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A REVIEW OF THE YELLOW INTERVAL DILEMMA

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Abstract—This paper deals with the evolution of rules concerning the setting of the yellow interval duration of a traffic light and the associated "amber (yellow) light dilemma". By employing realistic estimates of dynamical characteristics of vehicles, we investigate all possible options available to a driver facing a yellow signal. We show that the dilemma may be eliminated if the yellow interval duration is set according to a formula given by Gazis, Herman and Maradudin (GHM), for a given intersection geometry and an approach speed equal to the speed limit. We also show that a driver traveling with a speed less than the speed limit and being within his own dilemma zone from an intersection when the yellow indication commences, as defined by GHM, is able to clear the intersection if the yellow duration is set according to the formula given by GHM using the speed limit, but only by accelerating toward a speed equal to or higher than the speed limit. We comment on the redundant formulation of partitioning a yellow interval into a 'yellow' and a 'red clearance' interval. Finally, we present a brief historical review of the investigations and practices regarding the setting of the yellow (amber) interval duration, and conclude with recommendations concerning the setting and associated traffic ordinances. Copyright © 1996 Elsevier Science Ltd

INTRODUCTION

Ever since traffic lights were introduced in order to regulate movement along conflicting directions, there has been a need to regulate the transition between the various traffic phases. A yellow interval has been used to provide an orderly transition by clearing an intersection of one traffic stream before allowing another conflicting stream to proceed. Ground rules for the setting of this yellow interval were established, and published in the traffic engineering literature such as the Highway Capacity Manual (HCM). In parallel, traffic ordinances were developed regarding the obligations of drivers facing a yellow indication. Unless it is specified for other purposes, the interpretation of the 'yellow interval' in the following discussion coincides with the original definition of the amber signal and is equivalent to the current definition of an intergreen interval, or the sum of a 'yellow change interval' plus a following 'red clearance interval'.

Thirty-five years ago, Gazis, Herman and Maradudin (1960) (GHM) studied problems associated with the setting of the yellow interval. In their paper, GHM pointed out that incompatibility frequently existed between traffic ordinances concerning the obligations of a driver facing a yellow indication and the technical feasibility of compliance with such ordinances, based on constraints in the dynamical characteristics of the driver-vehicle complex. GHM defined the "Amber Light Dilemma" as a situation in which a driver may neither be able to stop safely after the onset of yellow indication nor be able to clear an intersection before the end of the yellow duration, and overall comply with the traffic ordinances. In what follows, we use the phrase 'yellow interval dilemma' in substitution of the 'Amber Light Dilemma' in order to conform to more recent terminology.

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Since the publication of the GHM paper, there have been numerous revisions of the HCM, and much discussion of the yellow interval dilemma in various papers. Unfortunately, some researchers and traffic engineers did not understand the dilemma correctly. Frequently, improper policies resulted from this lack of understanding. In general, in spite of a great deal of discussion, no satisfactory resolution of the dilemma has been attained. The GHM paper focused on identifying the incompatibility between the yellow duration setting and the related traffic ordinances. In this paper, we concentrate on proposing yellow duration settings which can eliminate the conflict. In addition, we extend the discussion to account for a broader range of vehicle characteristics than that covered in the GHM paper.

The yellow interval dilemma is an example of the incompatibility of man-made laws and physically attainable human behavior. When the yellow signal turns on, a motorist approaching a signal controlled intersection, may find himself too close to the intersection to stop, or to stop safely, and yet too far from the intersection to cross the clearing line in a manner compatible with traffic regulations, if the yellow interval duration is too short. In this paper, we attempt to clarify the problem of the yellow interval dilemma. Since quotations and misinterpretations of eqn (9) of the GHM paper have appeared in many different versions of the Institute of Transportation Engineers (ITE) handbooks and various research journals, we present a detailed discussion of the dilemma following the GHM paper. We then discuss how the dilemma can be eliminated through a proper design of the yellow interval and/or driver behavior. In certain instances, a driver may eliminate the dilemma by accelerating toward the posted speed limit, or higher. However, it would be dangerous to advise drivers to follow the proverbial interpretation of the yellow indication as an instruction to "accelerate with caution". For this reason, the only sensible approach is to provide a yellow duration consistent with the geometry of an intersection, and ensure the compatibility of the traffic ordinances with reasonable human behavior. We are limiting our discussion to the use of the yellow intervals in the United States, and the associated ordinances. A discussion of international practices would be interesting, but is beyond the scope of the present paper.

THE YELLOW INTERVAL DILEMMA

The discussion of this section follows that of the GHM paper. The notation has been changed to conform with that usually found in the ITE handbooks. Consider the traffic situation illustrated in Fig. 1, in which a motorist is moving at a constant speed v_0 toward an intersection. When the yellow indication commences and the driver is at a distance x from the intersection, he has two options: either to bring the vehicle to a full stop before the stopping line, or to clear the clearing line before the signal turns red. We will address the issue of introducing a red clearance interval later. We now assume a constant acceleration a_+ if the driver decides to cross the intersection and a constant deceleration a_- if the driver

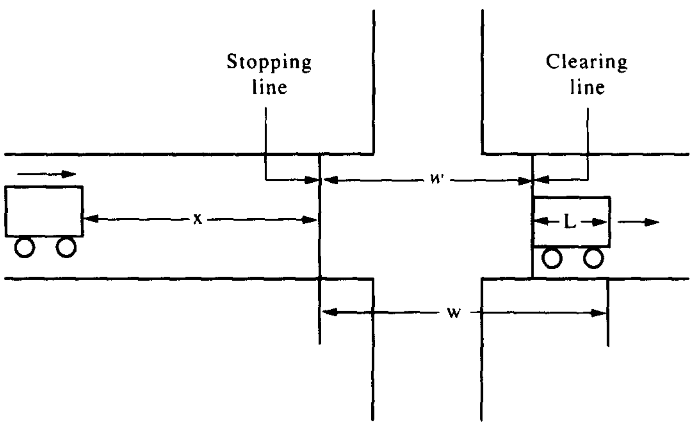

Fig. 1. The intersection under consideration showing the stopping line, clearing line, intersection width w, vehicle length L, and total intersection length $W = w + L$.

chooses to stop the vehicle. The acceleration or deceleration of the car starts after a time lag δ_+ or δ_-. This type of driver-vehicle perception–reaction time has been introduced in car-following platoon models (Chandler *et al.*, 1958; Herman *et al.*, 1959) and other literature. The width of the intersection is denoted by w, the length of the vehicle as L and the yellow duration as τ. If the driver comes to a stop without entering the intersection, we find that:

$$x - v_0\delta_- \geq \frac{v_0^2}{2a_-}, \tag{1}$$

where the left hand side (LHS) is the available stopping distance and the right hand side (RHS) is the actual braking distance for a given constant deceleration. If the driver can clear the intersection before the onset of the red phase, then

$$x + w + L - v_0\delta_+ \leq v_0(\tau - \delta_+) + \frac{1}{2}a_+(\tau - \delta_+)^2, \tag{2}$$

where the LHS is the distance to be cleared and the RHS is the distance that can actually be covered under the assumptions.

Next, we discuss the implications of eqns (1) and (2). Assuming the existence of a maximum acceptable deceleration a_- by which the vehicle can be brought to a stop before the stopping line safely and using eqn (1), we can define a critical stopping distance x_c as:

$$x_c = v_0\delta_- + \frac{v_0^2}{2a_-}, \tag{3}$$

within which the vehicle cannot be brought to a stop without running into the intersection. An acceptable deceleration is usually taken as $a_- \approx 0.3g = 10$ feet/s^2 (Gazis *et al.*, 1960). Notice that x_c is independent of the yellow interval duration. Assuming the vehicle moving through the intersection at a constant speed, it will be able to clear the clearing line if:

$$v_0\tau \geq x_0 + W, \tag{4}$$

where x_0 is the maximum distance the vehicle can be from the stopping line at the onset of the yellow indication and can still clear the clearing line at a constant approaching speed, and $W = w + L$. Thus, if $x_0 \geq x_c$, then the driver will be able to stop his car safely. However, if $x_0 \leq x_c$ and a driver-vehicle unit falls in a range $x_0 < x < x_c$, then the driver can neither bring his car to a full stop before the stop line nor clear the intersection before the yellow interval ends. A possible solution for the driver is to accelerate toward the intersection. However, the driver may violate the speed limit if his initial speed is already close to the speed limit. Thus, the range $x_0 < x < x_c$ is designated as a dilemma zone (Gazis *et al.*, 1960) and is illustrated in Fig. 2. The length of the dilemma zone is given by

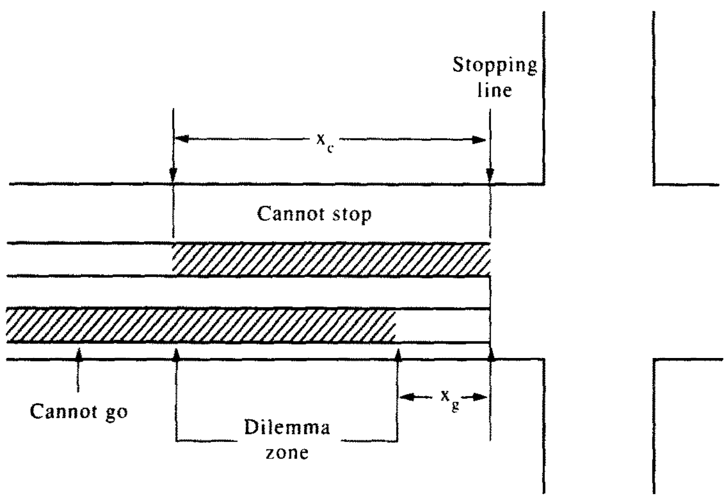

Fig. 2. The dilemma zone.

$$D = x_c - x_0 = v_0\,(\delta - \tau) + \frac{v_0^2}{2a} + W. \tag{5}$$

By letting D vanish, we find the minimum yellow interval duration for a non-accelerating vehicle as:

$$\tau_{\min} = \delta + \frac{v_0}{2a} + \frac{W}{v_0}. \tag{6}$$

Note that $\tau_{\min}$ depends on the initial speed v_0 of the vehicle, and it diverges for both small and large initial speeds. Hence, there is a minimum point for $\tau_{min}\,(v_0)$. The locus of the minima is plotted on Figs 3 and 4 of the GHM paper for different values of v_0 and W. Equation (6) does not always admit a solution for v_0 for an arbitrary τ. A simple manipulation leads to $\tau - \delta - \sqrt{2W/a} = (\sqrt{v_0/2a} - \sqrt{W/v_0})^2$, which means that if the yellow duration $\tau < (\delta + \sqrt{2W/a})$, there is a dilemma zone for all driver-vehicle units moving toward an intersection. However, if $\tau > \delta + \sqrt{2W/a}$, there are two solutions v_1 and v_2 for eqn (6) (see Figs 1–4 in the GHM paper). This implies that there is no dilemma zone for those driver-vehicle units moving with constant speed v in the interval $[v_1,v_2]$. Papacostas and Kasamoto (1992) have pointed out that a slow approaching vehicle will be in a dilemma zone, no matter how large the yellow duration is. This is true, but of no operational interest, since it is difficult to conceive that a normal driver will drive toward and through an intersection at such a low speed, except under some very unusual circumstances.

We next discuss the issue of partitioning the yellow duration into a new yellow and a red clearance interval. The time required to traverse the critical distance x_c by a vehicle moving at constant speed $v_0 \le v_l$ is $\tau_y = x_c/v_0 = (v_0\delta + v_0^2/2a)/v_0$, which is identical to the sum of first two terms in the RHS of eqn (6). Hence, to guarantee that all the approaching vehicles can enter the intersection safely, we may take $\tau_y(v_l) = \delta + v_l/2a$ as the new yellow interval, where v_l is the posted speed limit. Then, we are left to deal with the red clearance interval. Using the above chosen yellow time interval, and assuming no acceleration, we find that the time spent in the intersection after the onset of the red clearance by a vehicle with initial speed v_0 is $\tau_R = W/v_0-(v_l-v_0)/2a$ if $v_0(v_l-v_0)/2a \le W$; otherwise, the vehicle has passed the clearing line before the onset of the red indication. Then, the duration of the red clearance should be chosen as Max(τ_R), which is a diverging quantity as v_0 approaches zero. To obtain a meaningful result for the red clearance interval, we need to use a reasonable cut-off for the initial speed v_0; however the red duration cannot be less than W/v_l. Introducing a red clearance interval in the traffic operations is, in a sense, a redundant formulation to the dilemma problem. In the discussion which follows, we do not consider the partition.

The next question is whether a driver is able to clear the intersection before the onset of the red indication if he is at the critical distance from the intersection determined by his initial speed for an arbitrary yellow duration. There are two cases to be considered: (1) If the yellow duration is too short for a vehicle moving at a speed close to the speed limit v_l, then a driver will spend some time in the intersection during the red indication. Moreover, speeding up to a higher speed may not resolve this problem, as has been shown in the GHM paper. (2) If the yellow duration is set according to eqn (6) with v_0 replaced by v_l, then it can be shown that the dilemma zones for those approaching vehicles with $v_0 < v_l$ can be eliminated if the drivers accelerate toward v_l or a higher speed provided that the acceleration meets some criteria. In the next section, we elucidate the preceding statement.

YELLOW INTERVAL DURATION TAKING ACCELERATION INTO ACCOUNT

Consider a motorist who is driving toward an intersection with a speed less than the speed limit v_l, and is within the critical distance x_c from the intersection when the yellow signal turns on. If he were to accelerate to clear the intersection, the question is, will he make it? To answer this question, we need to know the time required for a vehicle to clear the intersection under these conditions.

Before discussing specific cases, we address this situation within a general framework. The following equations are analogous to eqn (2), and can be written for two different situations involving an arbitrary functional form of acceleration, for the critical distance case, namely:

$$x_c + W = v_0\delta_+ + \int_{\delta_+}^{\tau_a} v(t)dt + v_l(\tau - \tau_a),\ \tau \geq \tau_a \tag{7}$$

$$x_c + W = v_0\delta_+ + \int_{\delta_+}^{\tau} v(t)dt,\ \tau \geq \tau_a \tag{8}$$

where

$$\tau_a = \delta_+ + \int_{v_0}^{v_l} dv/a; \tag{9}$$

and $v(t)$ is the speed of the vehicle at time t. The quantity τ_a in eqn (9) is the time it takes to accelerate from an initial speed v_0 to the speed limit v_l, with acceleration $a(t)$ which may vary with time. The LHS of eqns (7) and (8) is the actual distance to be covered. The RHS shows the distance traveled by a vehicle that accelerates up to no more than the speed limit. Equation (7) describes case A, a situation in which a vehicle reaches the speed limit before passing the clearing line, while eqn (8) represents case B, a situation in which a vehicle crosses the clearing line before reaching the speed limit. The solution for τ given by eqns (7) and (8) is designated by τ_A or τ_B, respectively,

$$\tau = \begin{cases} \tau_A \geq \tau_a \\ \tau_B \geq \tau_a. \end{cases} \tag{10}$$

It should be noted that eqns (7) and (8) always have a solution for τ, which does not necessarily satisfy the inequality with respect to τ_a. If it does not, then the solution for τ_A or τ_B, respectively, is physically unacceptable. These situations are illustrated in Figs 3 and 4, for two different combinations of the parameters entering eqns (7) and (8). In Fig. 3, the solution for τ_A corresponds to $\tau_A < \tau_a$. In this case, the last term of eqn (7) is negative, meaning that the vehicle accelerates past the clearing line up to the speed limit, and then moves backwards at that speed to the clearing line, a physically unrealistic solution. The proper choice for τ is τ_B. In Fig. 4, on the other hand, the solution for τ_B given by eqn (8) corresponds to $\tau_B > \tau_a$, meaning that the vehicle continues to accelerate past the speed limit. In this case, the proper choice for τ is τ_A given by eqn (7). It may be noted that, under any circumstances, the solution τ_B given by eqn (8) is smaller than or equal to the solution τ_A. The two solutions are equal only when:

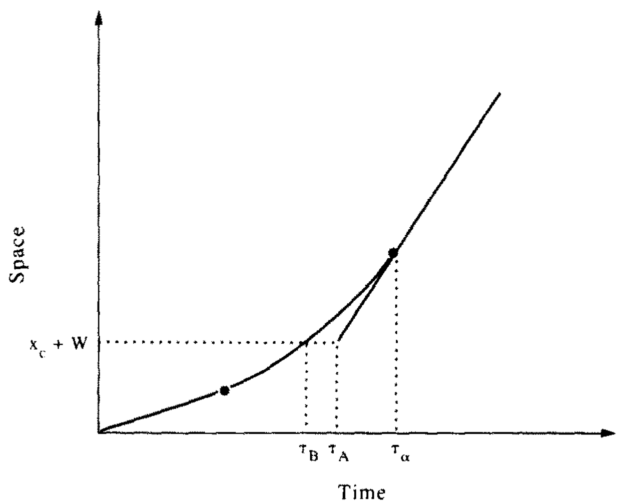

Fig. 3. A space–time diagram illustrating that $\tau = \tau_B$ is the acceptable physical solution.

$$\tau_A = \tau_B = \tau_a, \tag{11}$$

where the vehicle reaches the clearing line just as it reaches the speed limit. Figure 5 summarizes the above discussion, showing how the choice of the yellow duration τ changes with changing parameters. In this figure, s denotes the quantity $x_c + W$. When $s = s_B$ is relatively small, the vehicle reaches the clearing line before reaching the speed limit, and τ_B is the operative solution. When $s = s_A$ is relatively large, the vehicle must continue at the speed limit after reaching it, up to the clearing line, and τ_A is the operative solution. The two solutions are identical when $s = s_A = s_B$. We now discuss some special cases of the variation of acceleration with time.

Constant acceleration depending on the initial speed of a vehicle

For constant acceleration, on the basis of eqns (7) and (8), the time required for a driver to reach the clearing line by accelerating toward an intersection if he is at the critical distance from the intersection at the onset of the yellow indication, is determined by eqn (14) in the GHM paper, namely:

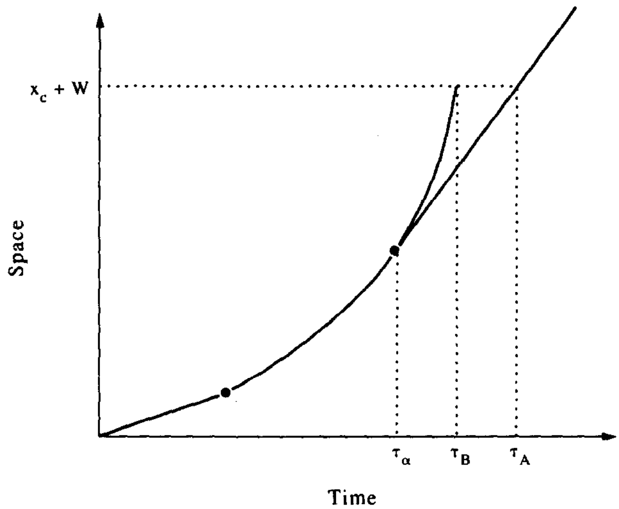

Fig. 4. A space–time diagram illustrating that $\tau = \tau_A$ is the realistic solution.

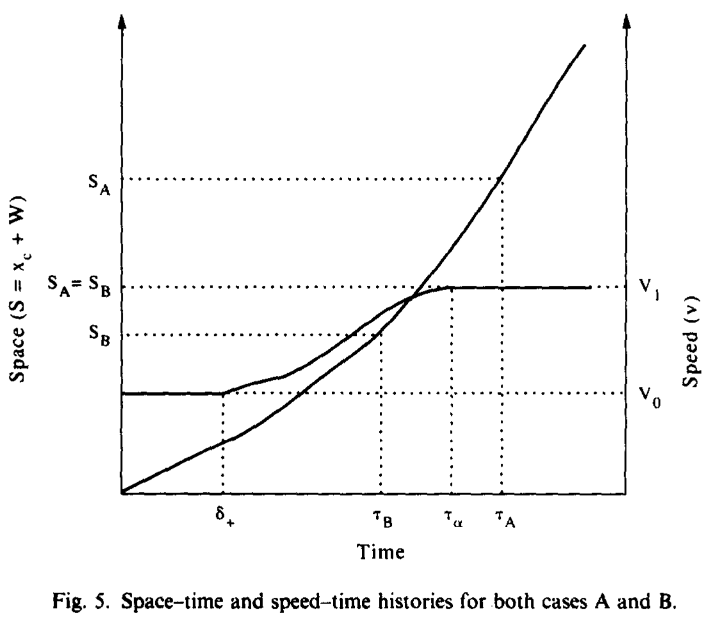

Fig. 5. Space–time and speed–time histories for both cases A and B.

$$x_c + W - [v_0\delta_+ + v_l(\tau_A - \delta_+ - \frac{v_l - v_0}{a_+}) + \frac{v_l^2 - v_0^2}{2a_+}] = 0, \quad \tau_A \geq \tau_a \tag{12}$$

$$x_c + W - [v_0\tau_B + a_+(\tau_B - \delta_+)^2/2] = 0, \quad \tau_B \leq \tau_a, \tag{13}$$

and

$$\tau_a = \delta_+ + \frac{v_l - v_0}{a_+}, \tag{14}$$

which are comparable to eqns (7)–(9). We now assume that the constant acceleration depends on the initial speed v_0 only and is zero above a critical speed $v_0 > V$. Following the GHM paper, we adopt a linear dependence of acceleration on the initial speed, namely:

$$a = \begin{cases} \alpha - \beta v_0 & \text{if } v_0 \leq \alpha/\beta = V \\ 0 & \text{otherwise,} \end{cases} \tag{15}$$

where α is the attainable acceleration at zero initial speed and taken as $\approx 0.5\,g$–$0.8\,g$ depending on vehicle type, the condition of the vehicle tires, pavement surface conditions, and the weather. Using eqn (3) for x_c and the notation $y = v_0/v_l$, we can solve eqns (12) and (13) and obtain:

$$\tau_A(y) = \delta_+ + y(\delta_- - \delta_+) + \frac{y^2 v_l}{2a_-} + \frac{W}{v_l} + \frac{v_l}{2a_+}(1 - y)^2 \tag{16}$$

and

$$\tau_B(y) = d_+ - y\frac{v_l}{a_+} + \left[\frac{2yv_l(d_- - d_+)}{a_+} + (\frac{yv_l}{a_+})^2(1 + \frac{a_+}{a_-}) + \frac{2W}{a_+}\right]^{1/2}, \tag{17}$$

with a_+ given by eqn (15).

It has already been shown that $\tau_A \geq \tau_B$. Therefore, in order to derive the maximum τ that is required, for any initial approach speed v_0, we need to investigate the dependence of τ_A on y. Next, we show that Max $(\tau) = \tau_A\ (y = 1)$. Differentiating eqn (16), we obtain:

$$\frac{d\tau_A}{dy} = \delta_- - \delta_+ + \frac{yv_l}{a_-} + \frac{v_l(1 - y)(\kappa y + \kappa - 2)}{2\alpha(1 - \kappa y)^2}, \tag{18}$$

where $\kappa = \beta v_l/\alpha < 1$. The value of this derivative at the end points, $y = 0$, and $y = 1$, is

$$\left(\frac{d\tau_A}{dy}\right)_{y=0} = \delta_- - \delta_+ + \frac{v_l}{2\alpha}(\kappa - 2) \tag{19}$$

$$\left(\frac{d\tau_A}{dy}\right)_{y=1} = \delta_- - \delta_+ + \frac{v_l}{a_-}. \tag{20}$$

It is reasonable to assume that δ_- and δ_+ are not much different, in which case the derivative is negative at $y = 0$ but positive at $y = 1$. Since $d^2\tau_A/dy^2 > 0$ for $y \in [0,1]$, the solution τ_A has only one minimum and no other extremes within the range (0,1). The value of τ_A at the end points of this range is:

$$\tau_A(0) = \delta_+ + v_l/2\alpha + W/v_l \tag{21}$$

$$\tau_A(1) = \delta_- + v_l/2a_- + W/v_l. \tag{22}$$

Assuming that the acceleration α is about equal to (but larger than) the maximum comfortable deceleration rate a_- (this is usually true), and $\delta_- \approx \delta_+$, we find that

$$\text{Max}(\tau) = \tau_A(y = 1). \tag{23}$$

According to the GHM paper, the end value of τ is the yellow duration that is required in order to eliminate the dilemma zone for a vehicle approaching an intersection at the posted speed limit v_l. It follows, from the above discussion, that if the yellow interval duration is chosen in this fashion, then any vehicle reaching the intersection at a speed lower than the speed limit can clear the clearing line by accelerating up to a speed equal to or higher than the speed limit. Remarks on the advisability of such a maneuver are given later in this paper.

Next, assuming constant acceleration given by eqn (15), we discuss the effect of intersection width on the required yellow duration τ, for two cases: (1) $v_l \le \sqrt{2\alpha W}$. In this case, $\tau_a(0) < \tau_A(0)$, which means that a vehicle with a zero-initial speed has reached the speed limit with the acceleration α prior to crossing the clearing line; and (2) $v_l > \sqrt{2\alpha W}$. In this case, $\tau_A(0) < \tau_a(0)$, which implies that τ_B is the operative minimum τ at speeds near zero, and τ_A is the one at speeds near the speed limit. Noting that the required acceleration time for a vehicle to reach the speed limit is $t(y) = \frac{v_l}{\alpha}\left(\frac{1-y}{1-\kappa y}\right)$ and the factor $\frac{v_l}{\alpha}$ is the time taken by a vehicle with zero initial speed to attain the speed limit, we see that the required acceleration time decreases as the initial speed of a vehicle increases, and the distance covered by a vehicle with an initial speed y over the time $t(y)$ is $S(y) = \frac{v_l^2}{2\alpha}\left(\frac{1-y}{1-\kappa y}\right)(1-y)$. For case 1, the quantity $S(y)$ is always less than W for $y > 0$, and equal to W only when $y = 0$. This means that a driver, initially within his own dilemma zone, will reach the speed limit before he crosses the clearing line, i.e. the required yellow duration for each individual speed class is given by τ_A for all y. For case 2, the quantity $S(y)$, minus the sum of the critical distance for a vehicle with initial speed y and the width of the intersection W, takes on a positive value for $y = 0$, and a negative value for $y = 1$. This implies that there exists an intermediate speed v_c such that vehicles with speeds $y < y_0 = v_c/v_l < 1$ cannot reach the speed limit before crossing the clearing line but vehicles with initial speeds $1 \ge y > y_0$ do. Hence, the required yellow duration for each speed class y is $\tau = \tau_B$ for $y < y_0$, and $\tau = \tau_A$ for $1 \ge y > y_0$.

Acceleration depending on the instantaneous speed of a vehicle

A refinement of the preceding section deals with the assumption that the acceleration of a vehicle decreases linearly with its speed, throughout the acceleration maneuver, i.e. $a_+ = \alpha - \beta v$. In this case, using eqns (7), (8) and (9), we can obtain τ_a, τ_A and τ_B as functions of y and other given parameters:

$$\tau_a(y) = \delta_+ + \frac{v_l}{\kappa\alpha}\ln\frac{1-\kappa y}{1-\kappa} \tag{24}$$

$$\tau_A(y) = \frac{x_c + W}{v_l} + (1-\kappa^{\;1})\tau_a + (\kappa^{\;1} - y)\,\delta_+ + \frac{v_l}{\kappa\alpha}(1-y) \tag{25}$$

$$\tau_B(y) - \delta_+ = \kappa\,[\frac{x_c + W}{v_l} - y\delta_+] + \frac{v_l}{\kappa\alpha}(1-\kappa y)[1 - \exp\left[-\kappa\alpha(\tau_B(y) - \delta_+)/v_l\right]]. \tag{26}$$

By repeating the procedures employed in the previous discussion and taking $\delta_+ \approx \delta$, we obtain the same conclusion in this situation, namely

$$\mathrm{Max}(\tau) = \mathrm{Max}(\tau_A) = \tau_A(1) \tag{27}$$

The above discussions are based on the linear relation between the acceleration and the speed of an approaching vehicle described by eqn (15). The conclusion that $\mathrm{Max}(\tau) = \tau_A(1)$ continues to hold if the acceleration of the vehicle lies above the straight line determined by eqn (15). However, if the acceleration of the vehicle drops below and deviates too much from the straight line, one may reconsider the above conclusion. In principle, for a given functional form of the acceleration, we can calculate the needed yellow duration using eqns (7)–(10) for each individual initial speed and search for the maximum value of the desired solution τ in the interval $y = [0,1]$ for setting up the duration.

MINIMUM REQUIRED CONSTANT ACCELERATION FOR GIVEN YELLOW DURATION $\tau_A(1)$

The yellow interval duration for a vehicle moving at the speed limit v_l is $\delta + v_l/2a + W/v_l$. It can also be long enough for the vehicles approaching the intersection with initial speeds less than v_l, if they are accelerating toward the speed limit or a higher speed with an appropriate acceleration. We may ask, what is the minimum acceleration required for a vehicle approaching the intersection when the yellow duration is set to accommodate a vehicle moving at the speed limit?

First, we note that for the given duration $\tau = \delta + v_l/2a + W/v_l$, there exist two solutions for v corresponding to a given a, $v_1 = v_l$, and $v_2 = 2a\ W/v_l$. These two solutions contract to one solution if $v_l^2 = 2a\ W$. We next discuss the required constant acceleration in case 1: $v_l^2 > 2a\ W$, i.e. $v_l > v_2$ and case 2: $v_l^2 < 2a\ W$, i.e. $v_l < v_2$, respectively. For case 1, a vehicle approaching an intersection with initial speed falling in the range $[v_2, v_l]$ does not have the dilemma, namely, it need not accelerate to clear the intersection. However, a vehicle with initial speed v_0 falling in the interval $[0, v_2]$ must accelerate in order to clear the intersection before the yellow duration ends. For the given yellow duration $\tau_A(1)$, using eqns (7) and (8), we find the accelerations for the vehicle for situations A and B as $a_A(y) = \dfrac{(1-y)a}{1 + y + 2a\ (\delta - \delta_+)/v_l}$ and $a_B(y) = \dfrac{2(1-y)[W - yv_l^2/2a]}{[\tau_A(1) - \delta_+]^2}$, respectively, where $y = v_0/v_l$. Hence the required acceleration for the vehicle is $a_r = a_A$ if $a_A > a_c = v_l(1-y)/(\tau_A(1)-\delta+)$, and $a_r = a_B$ otherwise, where a_c is the acceleration with which the vehicle reaches the speed limit with the given yellow duration. It can be shown in case 1 that the required acceleration is $a_r = a_B$ for $v_0 \leq v_2$ and $\delta \approx \delta_+$, and thus a_r drops from a maximum value $a_{max} = a_B(y = 0)$ to 0 as the initial speed increases from 0 to v_2. For case 2, all those approaching vehicles with initial speeds falling in the interval $[v_l, v_2]$ do not have a dilemma, but are violating the traffic regulations by being above the speed limit, while all those approaching vehicles with initial speeds less than the speed limit must accelerate in order to clear the intersection before the yellow duration ends. The expression for the required acceleration is exactly the same as the one given in case 1, namely, $a_r = a_A$ if $a_A > a_c$, and $a_r = a_B$ otherwise, but the acceleration drops from a maximum value $a_A(y = 0)$, which is $\approx a$ for $\delta \approx \delta_+$, to zero as the initial approaching speed of the vehicle increases from 0 to the speed limit v_l. It can be shown that, in both cases, $a_{max} \leq a$ for $\delta \approx \delta_+$. For case 1, the equality sign holds only when $v_l^2 = 2a\ W$.

In order to understand these results, we next present physical explanations. For case 1, one finds that at a very low speed the critical distance becomes negligible and the time it takes a driver to cross an intersection is simply the perception–reaction time plus the time a driver requires to cross the intersection with a constant acceleration a, namely, $\delta_+ + \sqrt{2W/a}$. Comparing this quantity to the given yellow duration, we notice that the driver can clear the intersection before the yellow interval ends only if $\sqrt{2W/a} \leq W/v_l + v_l/2a$ for $\delta \approx \delta_+$. Since the RHS of this equation reaches its minimum at $v_l^2 = 2a\ W$, the required acceleration takes on a maximum value a. A similar discussion applies for case 2.

The decreasing behavior of the required acceleration in the interval $[0, v_2]$ for case 1 or $[0, v_l]$ for case 2 can be understood. Consider two vehicles with selected approaching speeds u_1 and u_2, respectively, in one of the intervals, and $u_1 > u_2$. The time required for the vehicle moving with the higher speed u_1 to cross the clearing line is less than the time required for the vehicle moving with the lower speed u_2 [see Figs 3 and 4 in Gazis *et al.* (1960)]. If both vehicles accelerate with the same acceleration, then the faster vehicle will cross the clearing line earlier than the slower one. Hence, if they are crossing the clearing line simultaneously at the end of the yellow duration, the faster vehicle must have had a lower acceleration than the slower vehicle. This implies that the required acceleration decreases as the approaching speed increases in both intervals mentioned above.

The conclusion of the above considerations is the following: if the yellow interval duration is set according to $\tau = \delta + v_l/2a + W/v_l$, for a given speed limit v_l, then at the time the yellow indication commences, all driver-vehicle units with speeds $v \leq v_l$ and at their own critical distances (which are speed-dependent) from the intersection, can clear the intersection if they speed up with a constant acceleration larger than a_r. This does not mean that no drivers have a dilemma: in fact, the dilemma exists for those drivers approaching with speeds higher than v_l if $v_l > \sqrt{2a\ W}$.

The preceding discussion is independent of the nature of the vehicles facing a yellow light. However, the resulting minimum yellow interval will depend on the vehicle characteristics. In particular, long trucks with reduced acceleration capability would require a longer yellow interval. Whether or not traffic light should be designed to accommodate the most demanding vehicles depends on the expected frequency of such vehicles. If they are not accommodated, it must be understood that these vehicles may be forced to violate some ordinances, and sufficient warnings must be provided both to drivers of all vehicles and to the officers responsible for the enforcement of traffic ordinances.

REVIEW OF VARIOUS APPROACHES TO SETTING THE YELLOW INTERVAL DURATION

Various terms have been used in the literature to designate the yellow duration, such as amber period, clearance interval, amber phase duration, change interval, yellow clearance interval, and inter-green interval. The terminology becomes more confusing when one partitions the amber phase duration according to eqn (6) into a 'yellow' interval and an all-red clearance interval [see, for example, Chang *et al.* (1985), van der Horst and Wilmink (1985) and Sheffi and Mahmassani (1981)]. In the following discussion, we present a brief historical review of the formulations related to the yellow interval duration and the various interpretations of the yellow duration developed by different authors.

Matson's equation

As early as 1929, Matson (1929) put forward a formula for the clearance interval that would allow vehicles, crossing the stop line at the onset of the interval, to clear an intersection of width w. The required yellow duration was thus simply taken as the width of an intersection divided by the assumed constant vehicle speed, v, namely,

$$\tau = \frac{w}{v}. \tag{28}$$

In deriving this equation, Matson implicitly assumed an instantaneous response by the driver and an infinite deceleration capability by a vehicle close to, but not quite at the stopping line when the yellow indication came on.

Two decades later, Matson *et al.* (1955) proposed a more realistic formulation. Taking into account the driver perception–reaction time and denoting the time to stop a vehicle as t_1 and time to clear the intersection as t_2, they wrote:

$$t_1 = p + \frac{2S_1}{V} \tag{29}$$

$$t_2 = p + \frac{S_1}{V} + \frac{S_2}{V}, \tag{30}$$

where V is the initial approach speed of vehicle, S_1 the braking distance from an arbitrary position to the stopping line, and S_2 the width of the intersection not including the vehicle length. In our notation, eqns (29) and (30) are equivalent to equations $t_1 = \delta + \frac{v}{a}$ and $t_2 = \delta + \frac{v}{2a} + \frac{w}{v}$. By reasoning that, at high speed, the time it takes to stop a vehicle is longer than the time it takes to cross the clearing line, they claimed that the stop time is the critical value for the yellow duration. However, Matson *et al.* (1955) still did not come up with a reasonable operational approach.

1941 ITE Handbook

In this version of the handbook (Hammond & Sorenson, 1941), two formulae were suggested for the determination of a yellow duration. The first proposed formula was:

$$\tau = 0.8 + 0.04V + 0.7D/V, \tag{31}$$

where 0.8 s is the perception–reaction time, the vehicle deceleration taken as ≊ 18 feet/s^2 (a rather high deceleration), V the average speed of the approaching vehicles in m.p.h. and

D the width of an intersection in feet. The handbook further stated that "this length of the yellow light is the running time at average speed V between the point beyond which it is too late to stop when the yellow light comes on, and the far curb line". Equation (31) was first derived by Earl Reeder, who recognized that there exists a critical point, beyond which, an approaching vehicle cannot come to a safe stop when the yellow indication commences.

The handbook also quoted Matson's eqn (28) and proposed a second formula. Reasoning that a minimum safe stopping distance S was required by an approaching vehicle, the distance S was added to the numerator in eqn (28), yielding the duration

$$T = (w + S)/V, \tag{32}$$

where V is the speed of the clearing vehicle in m.p.h. Note that this equation is essentially the same as eqn (31) if S is interpreted properly. However, by reasoning that the cross-flow vehicles have a "free path" from the place where vehicles are stopped to the point where these vehicles would conflict with the clearing stream", a deduction term was introduced in eqn (32), namely:

$$T = (w + S)/V_1 - 0.682D/V_1, \tag{33}$$

where "D is the length of free path in feet to clearing stream, V_1 average speed of the accelerating cross-flow in m.p.h.", and 0.682 is a conversion factor. It may be noted that, for practical values of D and V_1, the correction term in eqn (33) is of little significance in terms of safety or time-saving.

1950 ITE Handbook

The 1950 version of the *Traffic Engineering Handbook* (Evans, 1950) quoted Earl Reeder's eqn (31) but changed notation, namely, $\tau = 0.8 + 0.04v_n + 0.07w/v_n$. Note that this formula again represents a high deceleration of about 18 feet/s^2. In general, this formula is the same as eqn (6), except that the variable w is the width of the intersection, and does not include the vehicle length. The *1950 ITE Handbook* also suggested a deduction to the yellow duration by subtracting the time taken by the lead stopped vehicle on the cross-street to accelerate from the stopping line to the point of conflict with the traffic stream. This suggestion ignores the possibility that a lead cross-street vehicle may be moving at a high speed at the onset of its green. However, similar ideas were suggested by others, many years later, without reference to the *1950 ITE Handbook* (Williams, 1977; Chang *et al.*, 1985).

1965 ITE Handbook

To calculate the time required for a vehicle to come to a safe stop, y_1, and the time required for a vehicle to traverse an intersection, y_2, respectively, the *1965 ITE Handbook* proposed the following two formulas (Baerwald, 1965):

$$y_1 = t + \frac{v}{2a}, \quad (sic) \tag{34}$$

$$y_2 = t + \frac{v}{2a} + \frac{w + l}{v}, \tag{35}$$

where l is the length of the vehicle, t the perception–reaction time, and a the deceleration taken as 15 feet/s^2. The second term of eqn (34) contains either a typographical or a computational error. The factor 2 in the denominator should be eliminated since it corresponds to covering the stopping distance at constant speed. The same error appears in subsequent papers by Williams (1977), Lin (1986) and others. To discourage driver disrespect of either too-short or too-long yellow lights, it suggests that yellow clearance intervals should be kept between 3 and 5 s. Where y_2 exceeds the value selected for the yellow interval and where hazardous conflict is likely, it suggests the use of an all-red clearance interval. It may be pointed out that the assumption that drivers take advantage of a long yellow interval has been observationally shown not to be correct in all cases (Olson & Rothery, 1962).

1982 ITE Handbook

In the 1982 edition of the *ITE Handbook* (Homburger, 1982), the critical distance x_c, the same as eqn (3) in the GHM paper, is used in concluding that the yellow interval required for a vehicle proceeding into an intersection is:

$$\tau_{\min} = \delta + \frac{v_0}{2a}. \tag{36}$$

This corrects the misinterpretation in the 1965 version of the *ITE Handbook*. Next, the author proceeded to quote eqn (22) in the GHM paper (same as eqn (6) in this paper) as giving the minimum clearance time for a vehicle crossing the intersection, implicitly ignoring the possibility of an accelerating vehicle crossing the intersection. The handbook repeats the recommendation of the 1965 version, regarding the desired range for the yellow interval and the use of an unspecified all-red clearance interval, where appropriate.

Further investigations

Following the GHM paper in 1960, studies concerning the yellow duration are split into three directions: (i) the studies based on the physical ideas in GHM such as those by Olson and Rothery (1962), and May (1968); (ii) proposals of other formulae to determine the duration with new parameters (Crawford & Taylor, 1961); and (iii) addition of terms to eqn (6) as those suggested by Williams (1977), Chang *et al.* (1985), Lin (1986), Lin and Vijaykumar (1988) and Lin *et al.* (1987), in order to shorten the yellow duration with deduction terms.

May (1968) studied the behavior of vehicles capable of various decelerations. His results indicate that the dilemma zone can be eliminated for those drivers who are willing to accelerate or decelerate their vehicles sufficiently to a higher or lower speed, i.e. the yellow duration is an acceleration-dependent quantity. The study would have been improved if a speed–time history and a location–time history were obtained after the onset of the yellow duration for each vehicle approaching the intersection; however, it is complicated and difficult to obtain such information for each individual vehicle.

Crawford and Taylor (1961) gave a formula based on measurements, obtained on a test track, of the critical distance needed to stop a vehicle moving at speed v and approaching an intersection. They proposed a yellow duration

$$\tau = 0.682[W/v + Kv^{0.6}], \tag{37}$$

where v is in m.p.h., W in feet, the number 0.682 is a conversion factor and K depends on the proportion of vehicles that have successfully stopped. However, one may question how well the test track experiments represent the actual driver experiences at traffic lights. In addition, no explicit relationship was given for the dependence of τ on the acceleration capability of vehicles.

The studies by Williams (1977) were motivated by trying to shorten the yellow duration. The proposed formula is the following:

$$\tau = \delta + v_{0.85}/2a + W/v_{0.85} - [\delta_+ + \sqrt{2d/a_+}], \tag{38}$$

where a_+ is maximum acceleration of cross flow traffic (16 feet/s^2), d the lateral distance at which a vehicle is in conflict with the cross-street traffic, and $v_{0.85}$ and $a_{0.85}$ are the 85th percentile of approach speed and deceleration, respectively. By adding the 'deduction term' to eqn (6), the author neglects the fact that the cross-street traffic can be at the posted speed limit at the onset of the cross-street green. In our opinion, it is not reasonable to compromise safety for the sake of a very small saving in yellow duration.

There was revived interest in the yellow duration problem in the 1980s. Stimpson *et al.* (1980) conducted experiments at two sites, in Maryland and Georgia, under various traffic conditions. In their experiments, a yellow interval was considered to generate potential conflict if the last vehicle to enter an intersection spent more than 0.2 s in the intersection after the onset of the red. In the Maryland experiments, they found that a yellow interval of 4.7 s generated a conflict level of about 19% during peak traffic and 15% during off peak traffic. By switching the interval to 6 s, the conflict was reduced to 2% at peak traffic and 1% at off peak traffic. At the Georgia site, the initial and final settings were about 4.3 and 5.7 sec, respectively. For the initial (final) setting, the conflict was found to be as high as 63% (19%), and 90% (21%) for off-peak and peak traffic, respectively. It is interesting to note that eqn (6) of this paper would yield, for the geometries and speed limits of the experimental sites and realistic assumptions concerning vehicle length and accelerations, a minimum required yellow interval of 5.1 and 6.1 s for the Maryland and Georgia sites, respectively. Van der Horst and Wilmink (1985) carried out field experiments in an urban network by extending the yellow time of each intersection by 1 s and found that the run-red offenses were halved, even after a trial period of 1 year.

Sheffi and Mahmassani (1981) developed a model of driver behavior at speeds of over 35 m.p.h. at isolated signalized intersections in order to assess the length and location of the "dilemma zone". A driver is assumed to stop if the time required to arrive at the stopping line at the onset of the yellow is smaller than some critical value. Under reasonable assumptions concerning the problem parameters, they used a probit calculation to determine dilemma zone boundaries. However, we note that this treatment does not appear to include consideration of the driver's other option of deciding to go and attempt to cross the clearing line within the yellow duration without considering the option of "accelerating with caution". Another study of driver behavior was that of Allos and Al-hadithi (1992) who used a Z-score discriminant model, including relevant variables affecting driver behaviors, to evaluate the success of some policies on the yellow interval.

Chang *et al.* (1985) adopted the following formula for the intergreen interval:

$$Y + AR = \delta_+ + \frac{v}{2a} + \frac{W}{v}, \tag{39}$$

where Y is the 'yellow' time interval, AR is the all-red interval and v is the 85th percentile of the approaching speeds. After collecting data over different sites, they proposed a fixed yellow duration of 4.5 s and a variable all-red interval, a policy likely to be acceptable for a relatively narrow range of geometry and approach speeds.

Other major studies were carried out by Lin and his collaborators (Lin, 1986; Lin *et al.*, 1987; Lin & Vijaykumar, 1988; Lin, 1992), and also by Wortman and Fox (1986). Lin and his collaborators studied the 'yellow' requirement over more than 20 different intersections, and proposed the formula:

$$Y = A + B\frac{W}{v}, \tag{40}$$

where v is the average speed of the traffic stream, and A and B are parameters obtained by linear regression to the data. Equation (40) suffers in that it calls for a regression analysis without a rigorous definition of the traffic conditions under which data is to be collected and without considering explicitly the geometry of the intersections and the driver-vehicle responses to the yellow indications. Lin *et al.* (1987) also proposed the formula:

$$Y = 4 + C, \tag{41}$$

where $-1 \leq C \leq 1$, but no clear indication is given for determining the numerical values of C for a given intersection. They suggested further that a universal yellow interval of about 4.5 s might be put into practice, although their observational data showed that the yellow duration varies by about 2 s over a large range of realistic conditions for satisfying a given percentile of drivers.

SUMMARY AND CONCLUSION

In the first part of this paper, we have re-examined the yellow interval dilemma by following the methodology in the GHM paper. It has been shown analytically that, for a yellow duration determined by eqn (6) with v_0 the speed limit, the dilemma zone associated with a vehicle traveling with a speed less than the speed limit when the yellow indication commences can be eliminated, provided that the driver accelerates with an acceleration larger than, or equal to, a required critical acceleration a_r, or accelerates with an available acceleration of the linear functional form given by eqn (15).

The next question is whether a driver should be advised to accelerate when approaching an intersection if he finds himself in a dilemma zone at the onset of the yellow duration. The most likely response from those responsible for setting traffic management policy will be that such advice is inappropriate. However, on the basis of our discussion, it must be recognized that some vehicles may have to accelerate in order to avoid violating traffic regulations. The important question then is how one should design the yellow duration in order to minimize the number of drivers caught in a dilemma zone, and the acceleration levels necessary to eliminate this dilemma zone.

A possible solution may be found by knowing the speed distribution function of the approaching vehicles. By making a cut-off at each tail of the distribution curve, we may compute the required yellow duration for the upper and the lower cut-offs and choose the larger one as the yellow duration. This duration should be large enough to allow those vehicles with low acceleration to clear the intersection before the onset of the red phase. The other solution is to compute the required yellow duration using eqns (7)–(10) for different initial speed classes of the approaching vehicles, and, by knowing from observations the accelerations of the vehicles in response to the yellow interval, select the largest value among the different classes as the yellow duration. In any case, it is advisable to modify the speed limit near an intersection, if necessary, in order to minimize the required minimum yellow duration given by eqn (6).

In view of our analysis, one may ask what procedure traffic engineering should follow in setting a yellow duration (or the length of a yellow interval plus a clearance interval in the 'new convention'). First, one needs to know what the $a_{\_}$ is for various vehicles and what the width of the intersection is. Note that $a_{\_}$ can be a weather-dependent quantity. Then, he may look for some historical data for other local intersections, e.g. the acceleration for each speed class of vehicles being in their dilemma ones when the yellow indication turns on, and see how drivers moving with speeds less than the speed limit respond to the yellow duration $\tau = \delta + v_l/2a_{\_} + W/v_l$. If one adopts the new convention, namely, breaking the original required yellow duration into two parts, one should choose the yellow and the red duration to be at least $\delta + v_l/2a_{\_}$ and W/v_l, respectively. One may consider extending the yellow duration if it appears short for some speed classes of drivers, depending on the circumstances and how often these situations occur. Shortening the duration to a value less than τ is not a good strategy since it can cause trouble for drivers approaching an intersection with a speed near the speed limit. For a highway intersection with posted lower and higher speed limits, a traffic engineer may determine the duration by calculating the duration time, τ, for the lower and higher speed limits, respectively, and then select the larger value as the yellow duration time.

Addressing the recent issue of dividing the yellow duration into a new yellow and a red clearance interval, we conclude that this approach is redundant to the yellow interval formulation suggested by GHM, since eqn (6) has included both a 'yellow' and a red clearance interval term. However, caution must be taken to ensure that vehicles waiting at a red indication along the cross-street do not "jump" the red, and also to ensure a proper interpretation of traffic ordinances by police enforcement personnel. We mention here that there is some success in dealing with the dilemma problem by using controller and detector technology, as has been reported by Zegeer and Dean (1978), Sackman (1977), and Parsonson *et al.* (1979). The main technique is in utilizing a green extension.

In the second part of the paper, we provided an overview of approaches to the yellow interval problem over the last five decades, including the work appearing in the ITE handbooks. It seems that not much progress has been made in this field during the last two decades. The reason appears to be that a longer yellow duration is viewed as undesirable because of the delay it causes to the traffic waiting along the cross-street. However, when insisting on using a short duration, one must combine it with a vehicle code that is compatible with the characteristics of drivers, vehicles, roads and signal operations, as has been suggested by GHM and others over the years.

Although the Uniform Vehicle Code (National Committee on Uniform Traffic Laws and Ordinances, 1968) and the recent ITE report (ITE Technical Council Task Force, 1994) allow approaching vehicles to enter an intersection during the yellow phase, they do not provide a solution for a driver in a dilemma zone. A red clearance interval prohibits all approaching vehicles to enter an intersection from all directions; however, if the red clearance interval is provided through a deduction from the required yellow duration given by eqn (6), as suggested by Chang *et al.* (1985) and Lin *et al.* (1987, 1988), then one must be certain that what is left in the 'yellow' does not force motorists to violate the traffic ordinances or face an unsafe situation. To be adequate, the 'yellow' and 'red clearance' intervals must be long enough to guarantee that most of the drivers approaching an intersection at a reasonable speed within the speed limit not only will be able to enter the intersection before the onset of the red clearance interval, but also will be able to clear the intersection before the red clearance ends.

The new 1992 Uniform Vehicle Code (National Committee on Uniform Traffic Laws and Ordinances, 1992) required that the cross-street traffic (or other conflicting traffic movements) yield to those vehicles that enter an intersection during the yellow interval, but are not able to cross the clearing line before the cross-street signal (or other signals for conflicting movements) turns green. This strategy makes the red clearance interval redundant conceptually. This is a good strategy, but can cause difficulties for some drivers. Consider a case in which a vehicle approaching an intersection collides with a cross-street vehicle at night. The driver may say that the cross-street vehicle did not slow down and he was in the intersection because the red clearance was too short; but the driver on the cross-street may claim that his opponent simply ran the red. With no witness or other information available, fair judgement would be virtually impossible.

From the above discussion, we see the complexities arising when we deal with the dilemma problem. The bottom line here is that it is important to provide an intergreen interval sufficiently long enough for drivers approaching an intersection in order to eliminate, or mitigate, the yellow interval dilemma. We already know that there are two solutions to the problem. One way is to mitigate the dilemma by reinterpreting the traffic rules as suggested by the *1992 Uniform Vehicle Code*, but this can lead to some of the aforementioned troubles. The other way is to set a long enough yellow duration (or, equivalently, the duration of a yellow interval plus a following red clearance in the new convention), and this needs some observations and proper enforcement. Of course, we may still argue that some pathological cases may exist in which the yellow duration is not 'long enough'; however, in the long run, we should really pay attention to how often these pathological cases occur and how drivers adapt to the existing yellow duration.

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