Misapplied Physics in the International Standards that Set Yellow Light Durations Forces Drivers to Run Red Lights

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The international standards that traffic engineers use to set yellow light durations are in opposition to the laws of motion. Misapplied physics creates systematic errors at signalized traffic intersections guaranteeing a steady stream of drivers running red lights. These errors are exploited by red light camera companies and governments. The systematic errors also induce thousands of vehicle crashes each year.

Many times we have approached an intersection when the light turns yellow and we did not know whether to stop or go. Sometimes we have accelerated to beat the light and other times we have slammed on the brakes in order to stop. Other times we have entered the intersection just a fraction of a second after the light turned red. Often we travel down the left turn lane and commit ourselves to enter the intersection, only to have the light turn to yellow and then to red before we could execute the turn.

These situations occur commonly to all drivers. We experience them many times a year. Over the decades we have grown accustomed and desensitized to such situations. These common red light running scenarios, though technically illegal, are the forced behavioral outcomes of systematic errors called dilemma zones, created by traffic engineers applying the Institute of Transportation Engineers' (ITE) Yellow Change Interval Formula.

A very small fraction of drivers run a red light each day because of dilemma zones. But nonetheless because there are hundreds of thousands of drivers traversing hundreds of intersections every day, that tiny fraction over a handful of years can accrue to the entire population of the city. This fact bears out in the red light camera data of Cary, North Carolina, population 135,000.

The ITE Yellow Change Interval Formula

Equation 1a is the Formula as it appears in ITE's *Traffic Engineering Handbook*[1] and *Traffic Signal Timing Manual*[2]. This Formula and its equivalents (1b, 1c) appear in traffic signal specifications for almost every jurisdiction in the world.

Equations 1. ITE Yellow Change Interval Formula

a	$$Y = t_p + \left[\frac{v}{2a + 2Gg}\right]$$
b	$$Y = t_p + \frac{1}{2}\left[\frac{v}{a + Gg}\right]$$
c	$$Y = t_p + \frac{1}{2}t_b$$

Variable	Description
Y	yellow light duration
t_p	perception/reaction time constant
v	vehicle's approach speed. The approach speed is not necessarily the speed limit.
a	safe comfortable deceleration constant of vehicle ITE's value = 10 ft/s^2 AASHTO's value[3] = 11.2 ft/s^2
G	Earth's gravitation acceleration
g	grade of the road in %/100. Downhill is negative grade.

a + Gg	effective deceleration of car
t_b	braking time. The time required by the vehicle to decelerate from **v** to a stop.

Because of the presence of the 2 in the denominator, the Formula cannot be an equation of motion describing general traffic. In physics terminology, a formula that is not an equation of motion means that objects in the real world do not abide by it. Had the formula been Y = t_p + v/(a + Gg), then the formula would be a proper equation of motion. But the Formula says v/**2**(a +Gg) which means that the yellow light lasts *half* the time it takes for a driver to stop. Because traffic engineers have been using this Formula for decades, what is the Formula's intent? Because the Formula is not an equation of motion, how does the Formula affect drivers?

The intent

Look at the Formula this way:

Eq 2. The Formula is Derived From Braking Distance

$$Y = t_p + \frac{\left[\frac{v^2}{2(a+Gg)}\right]}{v}$$

$$\textbf{Yellow Duration} = \textbf{Perception Time} + \frac{[\textbf{Safe Braking Distance}]}{\textbf{Approach Speed}}$$

In equation 2, the yellow light duration equals the time it takes for the driver to perceive and decide what to do when the light turns yellow, plus the time it takes for the driver to traverse the safe braking distance at the approach speed. The phrase "traversing the braking distance at the approach speed" mixes the physical properties of two different vehicles. The v in the numerator is for a decelerating vehicle. The v in the denominator is for a proceeding vehicle. There is an apple in the numerator and an orange in the denominator. The Formula divides apples by oranges. An algebraic fallacy. The quotient has meaning only for straight-though movement drivers but the fallacy always creates a side-effect called the dilemma zone.

For now regard the approach speed as the speed limit. We will take up the issue of approach speed versus speed limit later.

Let us define the *critical distance*. In equation 3, traffic engineers define the critical distance as the safe braking distance plus the distance the driver travels during the time that he perceives and reacts to the signal change to yellow[4].

Eq 3. The Critical Distance

$$\boldsymbol{c = v\, t_p + \left[\frac{v^2}{2(a + Gg)}\right]}$$

We are now ready to define the intent of the Formula. If the driver is farther from the intersection than the critical distance *c* when the light turns yellow, then he must stop. By embedding the braking distance into the yellow signal time, the Formula gives a faraway driver enough distance to stop safely, comfortably and legally. If the driver is closer to the intersection than *c*, then the driver does not have enough distance to stop comfortably or safely. The driver must proceed and enter the intersection. The Formula gives the proceeding driver enough time to enter the intersection before the light turns red *with the precondition that the driver approaches the intersection at a speed ≥ v.*

Forcing drivers to run red lights

The application of the Formula fails to properly apply physics in two respects.

1. The Formula never provides enough time for a driver to decelerate into the intersection. In order for the Formula to accommodate deceleration, the Formula must obey the equation of motion $a = \Delta v / \Delta t$. The Formula does not. The Formula shorts the required deceleration time by half. Therefore for any driver who must slow down anywhere within the critical distance before entering the intersection, the Formula creates a type I dilemma zone[5]. A type I dilemma zone is a region on the road where if the driver is in it when the light turns yellow, the driver can neither stop safely nor proceed safely without running a red light.

   Traffic engineers create type I dilemma zones at every intersection because every intersection must handle one or more of the following types of drivers:

a. Turning drivers. U, left and right turning drivers need to slow down to execute a turn.
b. Drivers going straight who must slow down for traffic waiting at the next nearby intersection beyond the immediate intersection. This situation is typical of busy downtown streets where intersections are close together.
c. Drivers going straight who must slow down for the stop sign or signal light at the intersection beyond the immediate intersection.
d. Drivers going straight who must slow down because the speed limit is less on the far side of the intersection.
e. Drivers going straight who slow down in preparation to change lanes because there is a fork in the road immediately after the intersection.
f. Drivers who tap their brakes to avoid colliding with vehicles entering or exiting business entrances or side streets close to the intersection.
g. Drivers who slow down before entering the intersection because of the high density of traffic within the intersection.
h. Drivers going straight who slow down to avoid colliding with an opposing left turning driver.
i. Drivers going straight who slow down for any objects in front of them.
j. Drivers who slow down for bumps in the road.
k. Drivers who slow down for potholes in the road.
l. Drivers who slow down to go over railroad tracks.
m. Defensive drivers. Drivers who slow down just to be cautious. No matter how defensive drivers are, they cannot escape dilemma zones[6]. In fact the more cautious the driver, the more the Formula forces the driver to run a red light.

2. The Formula assumes that all drivers know the precise location of the critical distance. If the driver guesses incorrectly by so much as an inch, deciding to go rather than stop, then the Formula will force him to run a red light. To compensate for a possible wrong guess, the driver often accelerates or slams on the brakes. The Formula is responsible for each behavior because the Formula does not provide the driver with a margin of error. In an instant the mandate to stop turns into the mandate to go. Because the Formula only provides half the time to stop, the driver is better off accelerating. Traffic engineers even expect drivers to accelerate[7]. The region on the road where a driver must guess whether to stop or go is called a type II dilemma zone[8]. A type II dilemma zone is different than a type I zone. Whereas a type I zone is a region on the road where the only outcome is running a red light, a type II zone is a region on the road where a viable solution exists, but the reasonably perceptive driver does not know what it is. Type II zones are also called indecision zones.

Engineers further misapply the Formula by . . .

1. Plugging in the wrong speed limit. The speed limit is 45 mph but the engineer accidentally plugs in 35 mph. In Cary, North Carolina, about 8500 drivers got flashed by a red light camera at an intersection whose yellow signal had this kind of mistake[9].

2. Plugging in 0% for the grade when the road goes downhill. 12,000 Cary, North Carolina drivers were flashed by a red light camera at this type of intersection[8].

3. Plugging in an approach speed which is less than the speed limit. This effectively forbids drivers from travelling at the speed limit. Drivers are entrapped by the speed limit sign. An approach speed set less than the posted speed limit shortens the braking distance below the minimum required by a driver travelling at the legal speed. The legally moving driver can no longer stop safely. Instead he must run the red light. Every protected left turn signal in Cary is like this, contributing to over 60,000 drivers running red lights[9].

4. Plugging in an approach speed which is less than the 85th percentile speed. For various reasons, engineers set speed limits artificially lower than the actual speed of free flowing traffic. This practice violates the engineering principle that design is supposed to accommodate human behavior, not oppose it. Every traffic signal in Cary is like this, contributing to 135,234 drivers running red lights[9].

5. Plugging in an approach speed measured at the stop bar.

6. Plugging in an approach speed measured only for queued vehicles in the left turn lane.

7. Plugging in a speed which is the average of the approach speed and the intersection speed for turning vehicles.

The size and location of type I dilemma zones is a function of approach speed, perception time, deceleration, grade, minimum intersection entry speed and actual yellow time[10]. On a level 45 mph road using the ITE standards, the dilemma zone in the left lane extends from 284 feet (critical distance) to 178 feet from the intersection. Any driver who in this zone travelling at the approach speed at the onset of yellow, who will enter the intersection at 31 mph or less, will be forced to run a red light[11].

History of the Formula

The Formula was invented in 1959 by Denos Gazis of GM Research Labs, Robert Herman and Alexei Maradudin. Gazis received his doctorate in engineering science and did his post-doctorate work in solid-state physics. Dr. Herman and Dr. Maradudin both received their doctorates in physics. Equation 4a is in Gazis' paper *The Problem of the Amber Signal Light in Traffic Flow*[12]. Equation 4b expresses the same meaning as 4a.

Eq 4. Gazis' 1959 Formula

A	$\boldsymbol{t_{min} = t_p + \left[\frac{v_0}{2a}\right] + \frac{w+L}{v_0}}$
B	$\mathbf{t} \geq \boldsymbol{t_p + \left[\frac{v_0}{2a}\right] + \frac{w+L}{v_0}}$

Variable	Description
t_{min}	minimum yellow duration
t_p	perception/reaction time
v_o	maximum allowable speed at the critical distance
a	safe comfortable deceleration of vehicle
w	Width of the intersection
L	length of the longest vehicle

Gazis explicitly designed the Formula to handle only one traffic situation. The Formula only handles the straight-through movement driver who can proceed unimpeded to and through the intersection at the maximum allowable speed[13]. That is the context of the Formula and that is as far as it goes. Gazis knew that his Formula was not a magic pill.

Gazis knew that it did not provide adequate time for vehicles that slow down before entering an intersection. He knew it neither worked for turning movements nor for vehicles at two close-by intersections[14]. He also knew that treating the Formula as an equality did not give the driver a margin of error. That is why Gazis expressed his Formula as an inequality.

- Today's traffic engineers misapply the Formula to every traffic situation.
- Today's traffic engineers misapply the Formula as an equality.

The third term $(w + L)/v_0$ in equation 4 is the amount of time it takes for a vehicle to travel across and clear the intersection at the maximum allowable speed. Today the third term is called the all-red clearance interval. It is the amount of time that drivers on all approaches see a red light. In Gazis' day, the all-red clearance time had to be added to the yellow light duration because the traffic signal hardware could not simultaneously display red on all approaches. This limitation is still true today for many traffic signals. Whether or not the traffic signal can show all-red, traffic engineers systematically take the third term out of context by setting v_0 to the maximum allowable speed instead of the speed of the slowest vehicle as it traverses the intersection. The slowest vehicle is usually the left-turning vehicle.

The 1959 Formula did not compensate for the acceleration due to gravity for vehicles on a hill. In 1982 ITE remedied that shortfall by including Gg in its *Manual of Traffic Signal Design*. The expression Gg is a small angle approximation. The approximation does not significantly affect the yellow time until grades exceed ± 10%. Not all jurisdictions use the version of the Formula with the Gg. Surprisingly California does not[15] and California includes San Francisco.

Approach speed

ITE instructs the engineer to plug in the *approach speed* for *v* into the Formula. Approach speed is a term specific to traffic engineering. Traffic engineers have a nebulous definition of approach speed. In the context of intersections, the approach speed is the speed with which a vehicle approaches an intersection.

Physicists are aware, however, that the definition of *v* in the Formula is not nebulous but exact. Approach speed *v* must be v_0, the *initial* velocity of the vehicle at the critical distance from the intersection. That is the physical meaning of *v* in the basic equation of motion *stopping distance* = $v^2/2a$.

But in 1965 ITE miscopied the original Formula into the *Traffic Engineering Handbook*[16]. v_0 became *v*. ITE forgot the naught.

Eq 5. ITE Traffic Engineering Handbook, 1965

$$Y = t_p + \frac{1}{2}\left[\frac{v}{a}\right] + \frac{w+L}{v}$$

The miscopy has led traffic engineers to believe they could define *v* arbitrarily. Since 1994 ITE has been instructing traffic engineers to set *v* for turn lanes to the *average* velocity of the speed limit and the vehicle's intersection entry speed[17]. Since 2004 the North Carolina Section of the Institute of Traffic Engineers (NCSITE) has been instructing the NCDOT to set v for turn lanes to the velocity measured for queued vehicles at the stop bar[18]. These practices are why yellow durations for left turn lanes are now 3.0 seconds while yellow durations for straight-through lanes are 4.5 seconds. The NCSITE practice causes red light camera citations to spike when Cary decreases left turn yellow durations from 4.0 to 3.0 seconds[19].

Speed limit

Approach speed is not necessarily the speed limit. Let us define speed limit.

Speed limit has a different meaning to the traffic engineer than to the judge, police officer and driver. To the traffic engineer, the speed limit is that speed which separates the bottom 85% from the top 15% of freely flowing vehicle speeds[20]. This method is called the 85th percentile rule. This method implies that the speed limit actually changes during the day and for different stretches of road. The 85th percentile speed during peak hours is less than that at midnight. The 85th percentile speed on a level part of the road is less than that going down a hill on the same road. The speed that engineers customarily post is the one they measured for a level road at peak-hour traffic. Engineers also round the posted speed to the nearest 5 mph.

Engineers purpose to set their speed limits by accommodating human behavior not by imposing iniquitous values. But because traffic engineers are restricted to handle wide variations of geography and human activity with a single blob of paint on a lonesome sign, the engineer's speed limit and what police and cameras think of as the speed limit are often at odds. As vehicles come down off a hill, a 35 mph sign at the bottom of a hill may be appropriate for the next section of road, but the 85th percentile speed of freely-flowing traffic at the speed limit sign may be 50 mph. The incompatibility spells opportunity for the assiduous police officer and the speed camera company.

While engineers are limited to express one speed limit for a road that requires many, engineers are not so limited when expressing the speed for setting yellow light durations. Engineers are mandated by their specifications to measure the approach speed independently from posted speed, compute the yellow duration from the approach speed, and set the traffic signal hardware to the result[21]. The approach speed must not be less than the posted speed limit lest it takes away the driver's legal right to travel at the speed limit. (Using an approach speed less than the speed limit disables a driver from stopping safely from the speed limit.)

Perception time and deceleration

The variance in measurements of perception time and deceleration contribute to dilemma zones as well. Traffic engineers misapply stochastic methods to compute an average perception time and deceleration for plugging into the formula. For example the 50th percentile perception-reaction time is 1 second yet it is known that valid perception-range times range from 0.6 to 2.4 seconds. Likewise for deceleration. It is known that 10 ft/s^2 is the average deceleration for a passenger vehicle but 8 ft/s^2 is the known average deceleration of a commercial vehicle. By misusing stochastic methods, traffic engineers plug in 1 second and 10 ft/s^2 and neglect more than half the drivers on the road. It is like an engineer designing a bridge to only withstand the average weight passenger car yet allowing school buses to traverse the bridge. Table 1 gives you an idea of averages used by different standards.

Table 1. The "Constants" Perception Time and Deceleration

	t_p	a
ITE	1 second	10 ft/s^2
AASHTO	2.5+ seconds	11.2 ft/s^2 – emergency stopping deceleration
Gazis/Original	1.14 seconds	10.7 ft/s^2
Commercial Driver License Manuals	2.5 seconds + 0.5 second air brake lag time	~ 8.2 ft/s^2

The American Association of State Highway and Traffic Officials (AASHTO) wrote an interesting chapter in *A Policy on Geometric Design of Highways and Streets* about driver reaction times[22]. AASHTO's conclusion is that "a brake reaction time of 2.5 s is considered adequate for conditions that are more complex than the simple conditions used in laboratory and road tests, but is not adequate for the most complex conditions encountered in actual driving".

Yet no jurisdiction uses AASHTO's recommendation. North Carolina uses 1.5 seconds. Oregon uses 1.7 seconds. Most others use ITE's 1.0 second.

Deceleration is also subjective. Comfortable deceleration means values around ⅓ G. Gazis' deceleration is ⅓ G. However Gazis said that ⅓ G is "feasible but is a fairly high deceleration not desirable in normal driving."[23] Many DOTs, like the NCDOT and the FWHA, use AASTHO's 11.2 ft/s^2. But AASHTO uses 11.2 ft/s^2 in the context of emergency stopping only. 11.2 ft/s^2 is not comfortable deceleration as required. Those DOTs expect drivers to slam on the brakes when they see a light turn yellow.

Note that the Formula does not consider commercial vehicles with air brakes. Air brakes do not engage all at once like passenger car brakes. Once the driver's foot presses the brake pedal, it takes about 0.5 seconds for the air pressure to build up so that the brakes can achieve a steady deceleration[24]. A traffic engineer desiring to cover the needs of all vehicles would add a brake lag time to the Formula but no engineer does this.

In the world of traffic engineering, the goals of traffic safety often compete with the goals of traffic flow. When push comes to shove, flow wins out. In the case of yellow light durations, the more the signal cycle spends in yellow phases, the less the signal cycle can devote to green phases. The more yellow, the less green. The less green, the less flow. Less flow is bad so engineers use values to cover the *majority* of drivers and vehicles, not values that cover *all* drivers and vehicles. So with willful intent, engineers design their signals knowing they will cause drivers on the wrong side of the percentiles or drivers of commercial trucks/school buses to run red lights. ITE explicitly recommends the practice of forcing drivers to run red lights. ITE instructs engineers to cap yellow durations to 5 seconds even when their own formula suggests they should be longer. ITE *hopes* that the all-red interval will allow the resulting red light runners to get to the other side of the intersection uninjured[25].

Gazis categorized red light runners into deliberate violators and non-violators[26]. Non-violators are red light runners entrapped by common ordinary and expected dilemma zone having to run the red up to 4.5 seconds into the red. Deliberate violators traverse the intersection in the middle of a red phase. Red light cameras and overzealous police officers do not discern the difference.

Consequences of the Formula

Yellow lights which are short by a fraction of a second relative to the Formula forced 400% more drivers to run red lights in Cary, North Carolina. Figure 1 is a graph[27] of the number of red light camera citations versus time at the eastbound approach on Cary Town Blvd. at Convention Drive. In March 2010, the Town of Cary fixed its incorrect assumptions about this intersection and increased the signal's straight-through yellow duration from 4.0 seconds to the Formula's 4.5 seconds. The number of red light runners decreased by about 75%. The Town of Cary had cut short this yellow since 1984. Cary placed a red light camera at this intersection in 2004. It was the first camera Cary installed.

Fig 1.

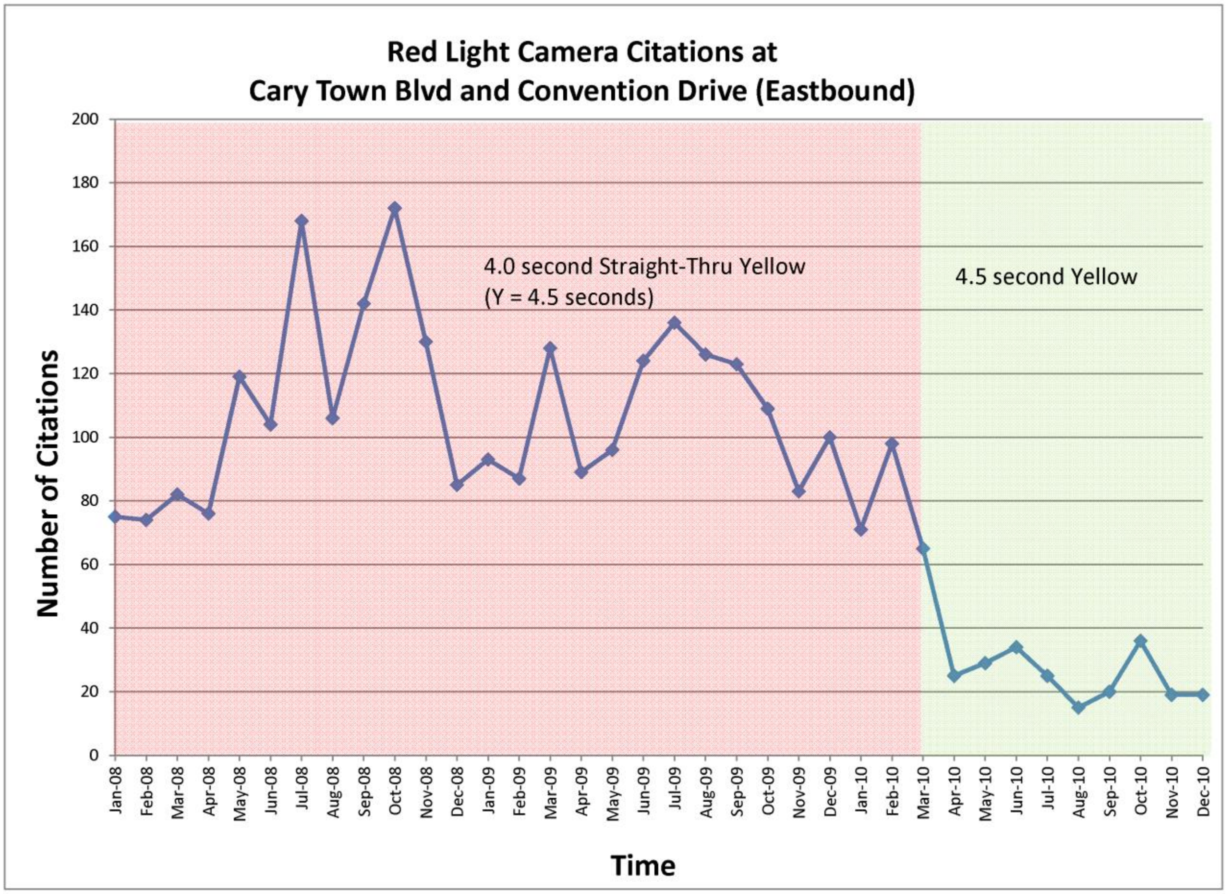
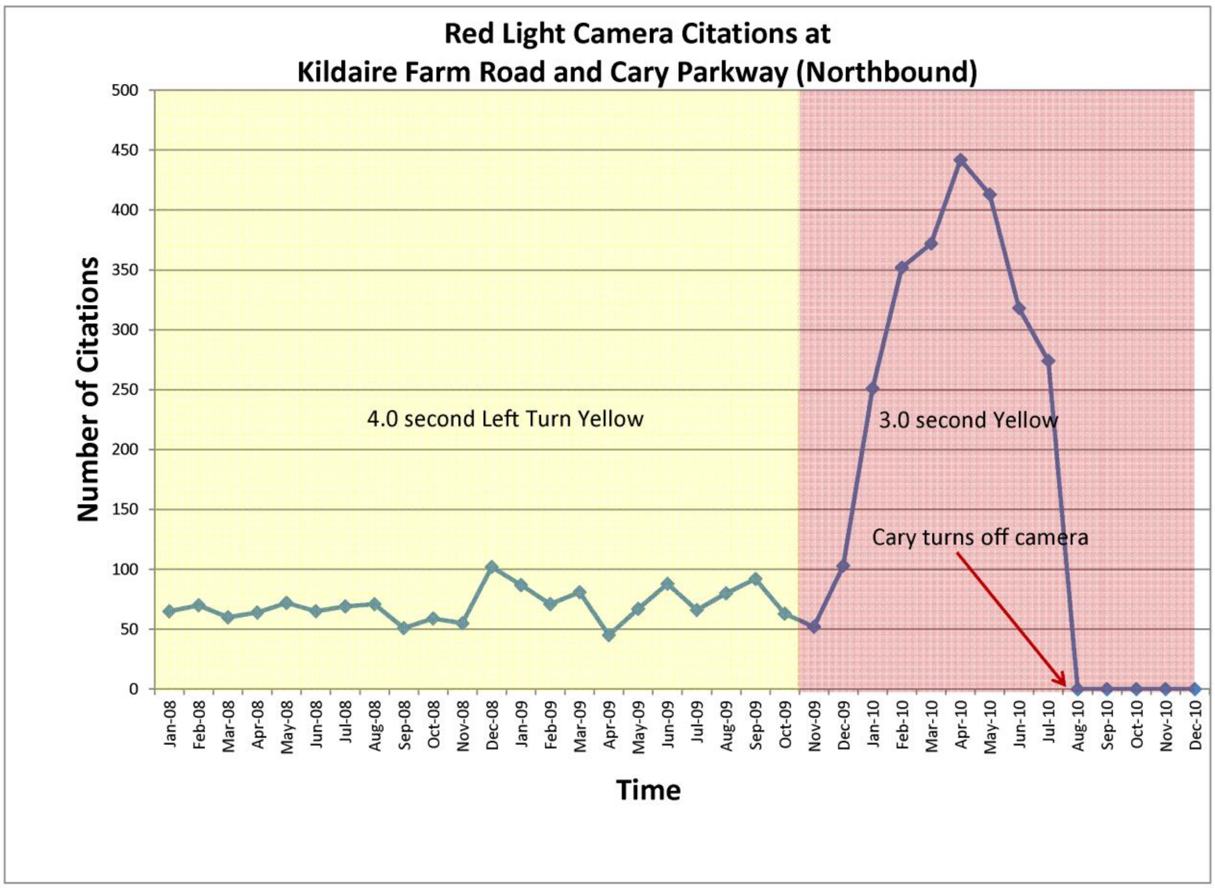


Figure 2 is a graph[27] of the number of red light camera citations on the northbound approach of Kildaire Farms Road at Cary Parkway. In January 2010, the Town of Cary decreased the left turn yellow duration from an already inadequate 4.0 seconds to 3.0 seconds using the NCSITE specification as justification. The Formula time for straight-through movement for this road is 4.5 seconds. The already high volume of red light runners increased about 600%. The Town of Cary turned off the camera in August 2010 for road repaving.

Fig 2.



Drivers running red lights during the low periods are not necessarily violators either. By simply applying the ITE Formula, the Town of Cary subjects certain drivers at all times to type I and type II dilemma zones. Reduction in the red light running rate only indicates a reduction in the sizes of the dilemma zones, not their absence. At Cary Town Blvd. and Convention, the low period red light runners are most likely type II dilemma zone victims before Cary does not record left-turn violations at this intersection. At Kildaire Farms Road and Cary Parkway, there always has been a type

I dilemma zone in the left turn lane because 4.0 seconds undercuts the laws of motion. Both intersections have a type II dilemma zone for straight-through traffic, and a type I dilemma zone for anyone who must slow down before entering the intersection.

The Town of Cary canceled its red light camera program in August 2012. Before that the Town of Cary had operated red light cameras at 17 approaches. Cary installed these cameras on the approaches that had the most numerous and longest type I dilemma zones. There was no exception.

Solution

The solution[28] is equation 6. Equation 6 handles most cases. It gives drivers the distance to stop. It gives drivers the time to proceed at the approach speed. It gives drivers the time to slow down into the intersection in order to execute a turn or avoid conflicts. In the Driver's Manual, the DMV can replace the meaning of the yellow light from "Yellow means that the signal is about to turn red, stop safely if you can, do not beat the light but we may make you run a red light anyway," to the instruction, "When seeing a yellow light, stop safely if you can otherwise proceed at the speed limit. You do not have to beat the light. If you need to decelerate before entering the intersection you can. The yellow light lasts long enough for you to brake to a stop. The worse that can happen is that you enter the intersection travelling slowly while the light is still yellow."

The solution does not mean that "yellow means stop." The solution means that when a driver needs to slow down into the intersection, he can now do so legally. The yellow light will still be yellow when he enters the intersection. The driver has only to understand that when he sees a light turn yellow, he can decelerate to a stop if he must, and he will not be penalized for it.

Today when a driver is near the critical distance from the intersection when the light turns yellow, if he decides to stop then the light will turn red *halfway* on route toward the intersection. The solution changes this. The light will turn red *when the driver arrives* at the intersection.

Many in the traffic engineering community baulk at the idea of such a long yellow. For a 45 mph level road, the yellow would be 7.4 seconds in North Carolina. Engineers express worries over highway congestion and fears over drivers disrespecting a long yellow light. The same concerns were voiced back in Gazis' day. Gazis and his colleagues dismissed those concerns with the riposte, "However, we believe that it is the duty of the traffic engineers and the drafters of traffic ordinances to present the average, honest, driver with a solvable decision problem[29]."

In the end the laws of physics do not really give an engineer a choice. Setting a yellow light less than equation 6 always guarantees that certain drivers will involuntarily run red lights. Equation 6 applies to all drivers, as opposed to ITE's Formula which only provides a solution to the subset of drivers who can guess exactly the location of the critical distance line and traverse the critical distance unimpeded at the speed limit or more.

Equation 6 eliminates dilemma zones. Both type I and II dilemma zones disappear altogether. The only reason why dilemma zones exist in the first place is because the ITE Formula mandates drivers to make a mutually exclusive decision with only one decision being the right one: The Formula presents, "Should I stop or should I go? Choose correctly or suffer the consequences." Remove the "2" from the Formula and then the driver always has the option to stop. The worse that can happen is that the driver begins to slow down and arrives at the intersection while the light is still yellow. The driver would then go slowly through the intersection on a yellow.

Equation 6 is not perfect either. It does not handle weather conditions. The technology does not yet exist to sense and transmit contributions by the weather to the vehicle's motion. The solution does not accommodate the force of the contribution by wind, or the contribution by water on the coefficient of friction between the road and tires.

Eq 6. The Solution

$$\boldsymbol{Y = t_p + \frac{v_0}{a + g\sin(\tan^{-1} G)}}$$

Variable	Description
Y	duration of yellow light
t_p	perception + reaction + air-brake time
v_0	velocity of vehicle measured at $v_0^2/2[a + g\sin(\tan^{-1}(G))]$ from the intersection $v_0 \geq$ posted speed limit
a	safe comfortable deceleration The value assumes that all vehicles from motorcycles to 18-wheelers have brakes which can exert a force to decelerate the

	vehicle at deceleration ***a***.
g	Earth's gravitational constant
G	grade of road (rise over run, negative values are downhill)
$gsin(tan^{-1}(G))$	precise expression for the contribution of Earth's gravity towards a vehicle's deceleration on a hill of grade G. When G > 0, use G = 0. When going up a hill, a driver presses his brakes less harder to achieve the same comfortable deceleration. When G < 0, the driver goes downhill but the act of pressing the brakes harder in itself feels abnormal. The driver becomes aware of the vehicle's braking ability. When $G < 0.10$, $gG \approx gsin(tan^{-1}(G))$.

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