

The Yellow Change Interval: Five Major Engineering Errors and Omissions



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Abstract

Engineering judgment starts with the proper application – not the misapplication -- of the physical sciences. This paper demonstrates the five major ways traffic engineers as a profession misapply science and math to yellow lights. Within a few years, the misapplications cause a handful of red-light cameras to issue more tickets than a city's population.

Under question is the math equation traffic engineers use to set the “yellow change interval.” It is called the “ITE yellow change interval equation.” ITE stands for the Institute of Transportation Engineers. The yellow change interval is the technical term for the duration of the yellow light. Many countries have adopted the ITE equation, but the equation is wrong. More precisely, misapplied. The use and misuse of this equation literally makes every driver unavoidably and systematically run red lights. The equation causes crashes by its physics. Its physics puts conflicting traffic in the intersection at the same time.

The ITE equation is not a federal standard. The ITE equation is not a federal guideline. The ITE equation is not in the MUTCD. The ITE equation is not in the MUTCDC (Canada). ITE itself does not recommend using the ITE equation. The equation is not an “ITE Recommended Practice”. ITE has been propagating this equation since 1965. Traffic engineers use this equation at their own discretion and personal liability as a licensed professional engineer.

The inventor of the equation, Dr. Alexei Maradudin, PhD. a physicist at UC Irvine has [rebuked ITE](#) and DOTs for misapplying his equation. ITE and traffic engineers see the symbols of the formula but do not fathom its physics. The physics of the equation conflicts with the physics of general traffic approaching an intersection. The conflict causes drivers to run red lights and red-light camera companies to make profit without end.

Engineers call the ITE equation the “kinematic equation”. “Kinematics” is the field of physics which studies the motions of objects without considering the forces acting upon them. Because traffic engineers do not know the kinematics of the formula, (1) Engineers apply the equation to vehicles which the kinematics oppose. (2) Engineers measure the equation’s input variables at a location contrary to the kinematics. (3) Engineers misapply stochastic methods thus computing input values which disallow the domain requirements of the legal motion of traffic and safety to be satisfied, (4) engineers misapply the analytic solution of emergency braking (a case which considers different forces) to the non-emergency use-case of vehicles approaching a traffic signal, and (5) engineers omit the calculation of tolerances, allowing law enforcement to punish millions of drivers entering the intersection within the uncertainty of the engineering.

Definition of Malfunction: Literally “bad function”. *mal* – function; *mal*: bad, badly, **wrong**

I. Traffic Engineers Misapply the Yellow Change Interval Equation

From the very beginning, traffic engineers use the wrong math equation to set the length of the yellow light. They use the following **malfunction** to set the yellow indication change interval **Y**, causing drivers to run red lights inadvertently:

$$Y = t_p + \left[\frac{v}{2a + 2Gg} \right]$$

From introductory physics, $t = v/a$ represents the time it takes for an object whose initial velocity is “v” to come to a stop. The yellow light equation divides that time by 2. **The presence of the 2 is the problem.** The yellow change interval is half the time it takes for a driver to stop his car. This one simple error is the cause of all dilemma zones, the presence of the red light camera industry, and the cause of the vast majority of crashes at signalized intersections.

By using the malfunction, [traffic engineers consider only drivers going straight through the intersection](#) in a permissive yellow law jurisdiction. The equation works only for this one special case. Only for this one case does the equation give drivers the distance to stop, and when drivers cannot stop comfortably, the time to reach the intersection before the light turns red. There are preconditions. One precondition mandates drivers who are too close to the intersection to comfortably stop, to travel at the speed limit or faster on route into the intersection never slowing down below the speed limit. Another precondition mandates that drivers must know the exact location on the road where stop turns into go. Math defines such a location; traffic engineers never mark the location. The driver must guess. The programmers of the red-light camera computer neglect the dynamics of guessing. The programmer literally frames the driver, in both the legal sense and the photographic sense, to make the driver look guilty, and then punishes the innocent driver for engineering which does not design for human factors.

The malfunction does not work for turning and impeded drivers. The algebra is wrong. The malfunction causes [turning](#) and [impeded drivers](#) to inadvertently run red lights. The malfunction shorts a left-turn yellow by at least 3 seconds on a 45 mph level road. The malfunction shorts a straight-

through yellow by at least 3 seconds on a 45-mph level road if the approaching driver has to slow down for a car entering the roadway from a business or side-street, a pedestrian or for any obstacle in front of him.

The correct function is:

$$Y_{mia} = t_p + \frac{v_c}{[a + \Gamma]} \quad \Gamma = \begin{cases} g \sin(\tan^{-1} G), & G < 0 \\ gG, & -0.1 < G < 0 \\ 0, & G \geq 0 \end{cases}$$

This correct function accommodates all allowable traffic movement. This function does not have the 2 in the denominator. This function is simply Newton’s Second Law of Motion. $F = ma$ where $a = v/t$. It allows for vehicles once too close to the intersection to stop, to slow down to turn or to slow down for a hazard and still be able to enter the intersection legally. As opposed to the malfunction, the correct function never creates a dilemma zone because it *always* provides the driver with the option to stop comfortably without running a red light.

II. Traffic Engineers Use the Wrong Velocity “v”

Traffic engineers use the **malfunction v()** to set the velocity of turning vehicles approaching the intersection.

$$v = 20 \text{ mph, for protected left and right turn lanes}$$

Traffic engineers in most jurisdictions use approximately 20 mph to plug into the Y malfunction for protected left or right turn lanes. By setting v to 20 mph, the traffic engineers give all approaching drivers the stopping distance of a 20 mph car. That is 90 feet or 5 car lengths.

Traffic engineers will set v to 20 mph even when the speed limit is 45 mph. In order to stop within 90 feet at 45 mph is physically beyond the emergency braking capabilities of any vehicle.

Engineers misapply the velocity malfunction into the ITE yellow change interval equation. The ITE equation does not work for turning vehicles. Engineers therefore make a double error. Wrong velocity. Wrong equation.

The flip side of the equation. The equation is not only about having the distance to stop, but also about having the time to reach the intersection once one cannot comfortably stop.

Engineers and red-light camera companies blame drivers for speeding when they see a yellow. But the laws of physics require a driver to speed. Let us set up a simple story problem.

How fast must a driver go to legally enter an intersection? Traffic engineers typically set a left turn yellow to 3 seconds even on a 45-mph road. The stopping distance for a 45-mph sedan is about 300 feet. A driver is approaching the intersection. The driver sees a green arrow. There are no cars in front of the driver. There are no cars queued to turn left. Driver is going 45 mph, the speed limit. Once the driver is within 300 feet, he is too close to stop and must proceed into the intersection. Light turns yellow. Yellow is 3 seconds long. Driver has 3 seconds to traverse 300 feet. Rate = distance / time. Driver must proceed at 300 feet / 3 seconds = 100 ft/s = 68 mph to reach intersection before light turns red. Who caused the driver to speed? Who caused the driver to beat the light? The engineer did. By setting the yellow to 3 seconds, the engineer has caused the driver to break the speeding law.

ITE wrote in its Traffic Engineer Handbook [that some drivers must beat the light.](#)

The correct function for determining approach speed is to measure the velocity of freely-flowing vehicles at the critical distance upstream from the intersection. “Safe and comfortable stopping distance” is a synonym for critical distance. Physics defines “v” as measured at the critical distance.

$V = V_c$ 85th percentile speed of freely-flowing traffic, the speed measured at the critical distance c .
 $V_c \geq$ speed limit

The correct function for the location of the critical distance c is:

$$c = t_p v_c + \frac{v_c^2}{2[a + \Gamma]} \quad \Gamma = \begin{cases} g \sin(\tan^{-1} G), & G < 0 \\ gG, & -0.1 < G < 0 \\ 0, & G \geq 0 \end{cases}$$

Setting v lower than the speed limit for any lane denies the law-abiding driver the distance necessary to stop. All drivers are allowed to go the speed limit regardless of lane.

III. Traffic Engineers Misapply Stochastic Methods.

A stochastic method is a mathematical treatment restricted to random events. A stochastic method is usually some statistical method like averaging. As an example of a proper application of a stochastic method, go out to a signalized intersection and measure the duration of a yellow light. You have a stop watch. Assume that the duration of the yellow light is the same length for every light cycle; that is, the yellow change interval is *constant*. What is not constant, but rather is random, is your timeliness to press “go” and “stop” on your watch. Your first measurement of the yellow light duration is 4.2 seconds. The second measurement is 4.4 seconds. Then 4.3, 4.4, 4.2, 4.6, etc. To compute a value,

you average these random measurements. Then you compute the standard deviation for the accuracy of the average. It will be something like 4.3 +/- 0.2 seconds. Because the yellow light's duration is a *constant* and that your measurement ability is *random*, it is proper to use stochastic methods like averaging and computing standard deviations.

Misapplications of Stochastic Methods

- a. Example 1. A structural engineer designs a bridge to sustain only the average weight passenger car but he allows school buses to cross the bridge. (The weights of cars are not random; therefore, the structural engineer cannot consider only the average weight car.)
- b. Example 2. A BBQ chef considers the temperature which he cooks steaks: 145°F. 145°F kills the bacteria in the steak. The chef considers the temperature he cooks chicken: 165°F. 165°F kills the bacteria in a chicken. The BBQ chef then computes the average temperature: 155°F. From then on, the BBQ chef cooks all meats at 155°F. When the chef invites you over for a chicken dinner, are you going to accept the invitation?
- c. The traffic engineer is the BBQ chef. Traffic engineers assert in practice that perception-reaction time and deceleration are *universal* constants—that one value applies to all traffic. That is false. One value does not apply to all traffic. There is not a single perception-reaction time which applies to all drivers. There is not a single value for deceleration that applies to all vehicles. The values traffic engineers assert do not cover the drivers and vehicles *allowed* on the road.
 - i. A grandmother will take 2.5 seconds to perceive and react to a yellow light. You may take 1.2 seconds. A video gamer may take 0.7 seconds. Which driver perceives correctly? Asking such a question, let alone answering it, is invalid. Reality tells us that there is known *range of equally-valid* perception-reaction times. (See Gates' research below.) But traffic engineers misapply a stochastic method by computing the average for a passenger car driver. Engineers assert "1 second".
 - ii. Traffic engineers believe that an 18-wheeler comfortably decelerates as rapidly as a Toyota Corolla. We know comfortable deceleration for a commercial vehicle is about 8 ft/s². We know from the FHWA that a bus that decelerates at 7.4 ft/s² will throw a standing passenger to the floor. But traffic engineers set "a" to 10 ft/s² which is the average comfortable deceleration of a passenger car on dry pavement.

Traffic engineers use **malfunction $t_p()$** to set the perception-reaction time:

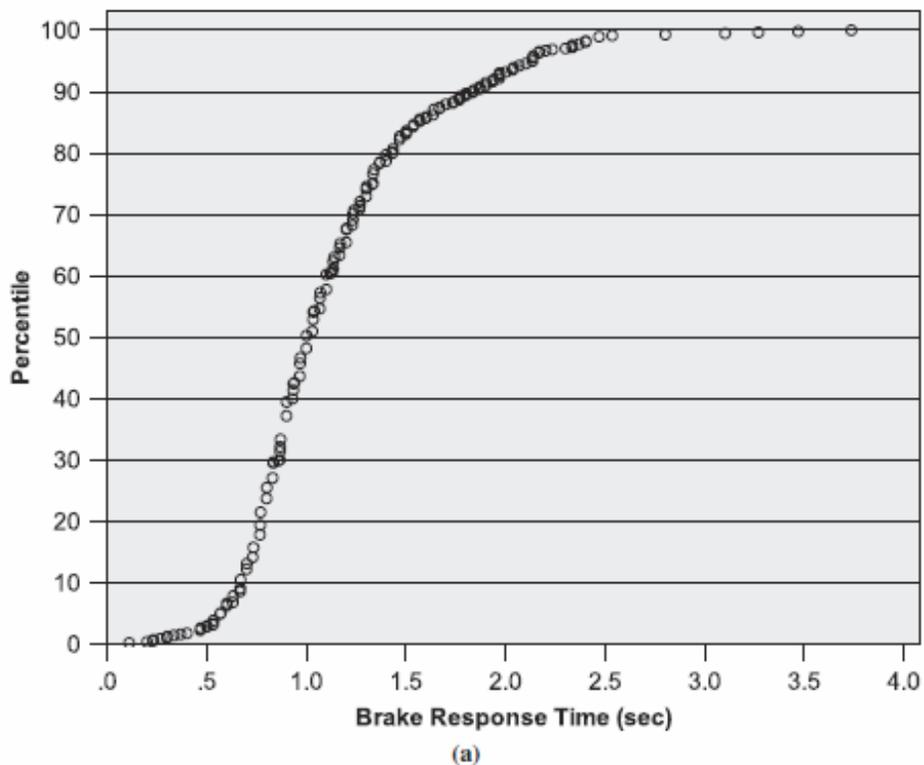
$$t_p = \mathbf{50th\ percentile}, \text{ typically } 1 \text{ second}$$

Traffic engineers use the 50th percentile perception-reaction time among passenger car drivers driving on dry pavement for the simplest intersection. In so doing, traffic engineers knowingly force over half the driving population to run red lights.

The correct function for perception-reaction time is:

$$t_p = \text{max PR time, typically 2.5 seconds}$$

That is what the following graph of the empirical data shows. The graph is from [Gates, Dilemma Zone Driver Behavior as a Function of Vehicle Type, Time of Day and Platooning, Transportation Research Record: Journal of the Transportation Research Board, No. 2149](#), Transportation Research Board of the National Academies, Washington, D.C., 2010, .p. 87.



Gates is measuring the P-R time by measuring how long it takes from the light turning yellow to the brake light coming on. “Brake-response” is the way traffic engineers measure “perception-reaction” time for braking scenarios.

Within the graph are both passenger and commercial vehicle drivers. Commercial drivers as a demographic require more P-R time, not less, than passenger car drivers.

TABLE 3 Brake Response Time Descriptive Statistics and Results of Statistical Analysis

Factor	Level	Count	Mean (s)	SD	Percentiles (s)		
					15th	50th	85th
Vehicle type	Car	315	1.17	0.50	0.77	1.03	1.64
	Light truck	226	1.08	0.46	0.70	0.97	1.47
	Single-unit truck	23	1.17	0.50	0.59	1.10	1.65
	Tractor trailer	8	1.18	0.61	0.57	1.02	2.13
Time of day	Peak	228	1.17	0.51	0.77	1.07	1.61
	Off-peak	344	1.11	0.47	0.72	1.00	1.54
Platoon	Platooned	185	1.14	0.47	0.77	1.03	1.60
	Not platooned	387	1.13	0.49	0.73	1.00	1.57
Speed			Not applicable				
Travel time to intersection			Not applicable				
Deceleration rate			Not applicable				
Full model ^b	All data	572	1.13	0.48	0.73	1.00	1.57

Traffic engineers assume that commercial drivers need less P-R time than a passenger car driver, citing without proof that “commercial drivers are more experienced.”

In addition to more P-R time, commercial truck drivers require 0.5 seconds for air-brake pressurization time. Traffic engineers neglect that requirement too.

In addition to the deceleration malfunction, the perception-reaction time malfunction is a reason why all jurisdictions puts in danger a disproportionate number of commercial vehicles. You can see this in Suffolk County’s [red light camera videos](#). While only 1 in 100 vehicles is a commercial vehicle, half of the clips in Suffolk County’s video show commercial vehicles running the red lights. The video clearly demonstrates Timothy Gates’ conclusion.

Traffic engineers use malfunction a() to set the “safe and comfortable” deceleration of a vehicle.

$$a = \text{50th percentile, typically } 10 \text{ ft/s}^2$$

When $a = 10 \text{ ft/s}^2$, traffic engineers use the 50th percentile safe and comfortable deceleration for a passenger car. The smaller the value for a , the slower the vehicle’s comfortable deceleration.

The correct function is this:

$$a = \text{min, typically } 7.0 \text{ ft/s}^2$$

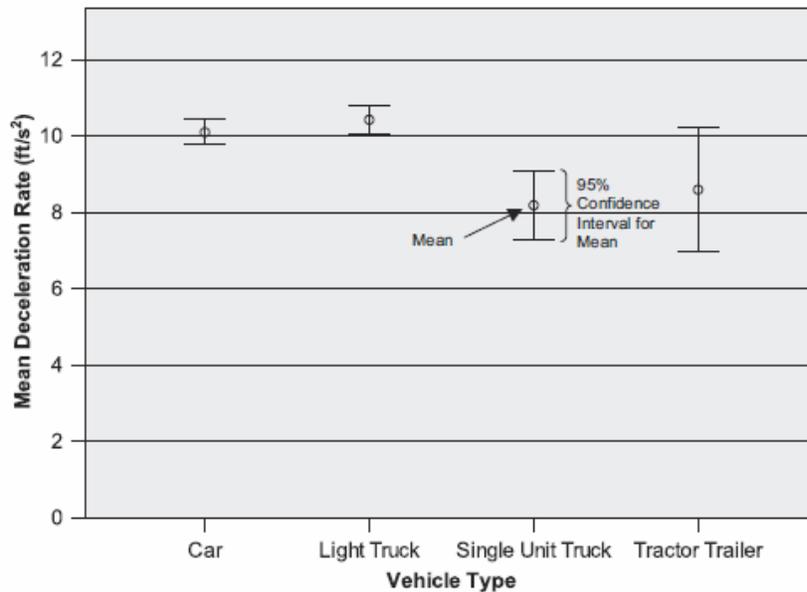


FIGURE 4 Mean and 95% confidence interval for deceleration rate by vehicle type.

IV. Traffic Engineer Misapply the Analytic Solution of Emergency Braking to Non-Emergency Approach to a Traffic Signal

$$Y = t_p + \left[\frac{v}{2a + 2Gg} \right]$$

The problem here is that traffic engineers have a “+ 2Gg” in the denominator. The “+2Gg” is incorrect. This error shorts the critical distance for vehicles ascending a hill, thus shorting the yellow change interval. The driver does not expect a shorter light when he ascends a hill and so he inadvertently runs the red.

It critical error made here is that traffic engineers do not discern the difference between analytic and physical solutions. A physical solution is a mathematic equation which manifests itself in nature. An analytic solution is a mathematic equation where the math works, but the math does not necessarily manifest itself in nature.

In 1982 Kell and Fullerton (K&F) introduced the grade extension +2Gg to Gazis’ yellow change interval equation. K&F’s extension is an analytic solution. It is not a physical one. K&F’s error is that they extended the mathematics from the stopping sight distance (SSD) equation--whose mathematics applies only to emergency stopping, to the yellow change interval--whose mathematics applies to non-emergency stopping. The grade mathematics of the SSD apply only when a vehicle’s maximum braking ability has been reached. Only when reached does gravity contribute to the acceleration/deceleration of the vehicle as the SSD equation describes.

For vehicles approaching a traffic signal on a hill, the dynamics differ than those assumed in the SSD equation. Dynamics are the forces causing the vehicles to move. When the dynamics change, the kinematics change.

Unlike for emergency stopping, the driver ascending a hill towards a traffic signal is not concerned with his maximum braking capability. The driver does not intend to slam on the brakes and have his air bag deploy. The driver is only concerned with how comfortable his body feels decelerating. To achieve a comfortable deceleration, the driver presses his brake less. The resulting deceleration as if he is decelerating on level ground. This shows in the data. The result is the same critical distance. While the critical distances are the same, the yellow change intervals are different. The yellow change interval is the time to traverse the critical distance. When ascending a hill, that time is a little longer than that of a level approach. The driver is generally not aware that gravity is decelerating him. If the driver watched his speedometer, he would notice that his speed is decreasing as he moves toward and into the intersection. Gravity. The ITE equation always short the yellow for any vehicle decelerating into the intersection.

Descending a hill towards a traffic signal is similar to emergency stopping. The resulting 2Gg works in both cases. It works for similar and different reasons. As with emergency stopping, the driver is concerned with how hard he must hit his brakes. In both cases, the driver compensates for gravity. The limiting factor for emergency stopping is the frictional forces between tire and pavement. The limiting factor for a non-emergency stop to a traffic signal is how comfortable he feels he can apply his brakes. For the driver, pressing the brakes harder than normal is never comfortable. When doing this, the driver looks into his rear-view mirror to see whether he will be rear-ended.

V. Traffic Engineers Omit the Calculation of Engineering Tolerances

The omission results in law enforcement punishing drivers for engineering error.

Red light cameras expose this error. Red light cameras enforce the imprecise yellow change interval calculation to precision. A traffic engineer will set a yellow change interval on a 45 mph level road straight-through lane to 4.3 seconds. But the fully-qualified mathematical value is 5.3 +/- 2.2 seconds.

For a 4.3 second set yellow light, the delay-time **D** set for the red light camera system should be $(5.3 + 2.2) - 4.3 = 3.2$ seconds.

The **malfunction D()** is used to set the delay-time for red light camera system.

$$D = 0 \text{ seconds}$$

Or similar to . . .

$$D = 0.3 \text{ seconds}$$

A red light running event will not occur unless the driver enters the intersection after time **D** has passed since the light turned red. Law enforcement arbitrarily sets these values but these values can be computed by engineering practices.

One should calculate the delay time for a straight-through lane using:

$$D = \Delta t_p + \frac{v_c}{2a^2} \Delta a$$

And for a turn lane:

$$D = \left| \frac{2v_c}{v_c + v_e} \Delta t_p \right| + \left| \frac{v_c^2}{a^2(v_c + v_e)} \Delta a \right| + \left| \left(\frac{2v_c \left(t_p + \frac{v_c}{2a} \right)}{(v_c + v_e)^2} \right) \Delta v_i \right|$$

The correct D functions compute a delay of 2.2 and 3.4 seconds respectively for a 45 mph level road. Because there are uncertainties in the constituent values for perception-reaction time, deceleration and intersection entry velocity, the uncertainties propagate to the yellow change interval.

History and Why

The errors and omissions began in 1965 when the Institute of Transportation Engineers miscopied the yellow change interval formula from a General Motors science paper into its own Traffic Engineering Handbook. The Handbook omits the provisos, preconditions, warnings and restrictions on the formula. The Handbook even omits the crucial subscript “0” from v_0 which to this day still leads traffic engineers to invent arbitrary locations to measure the approach speed.

Traffic engineers are reluctant to increase the yellow change interval. The main reason is they believe that increasing the yellow reduces traffic flow. This justification was debunked in the original General Motors paper (1959). Traffic engineers also believe that increasing the yellow will cause more drivers to disrespect the yellow and run more red lights. The disrespecting the yellow argument was formally debunked in 1961 in a different paper written by Olsen and Rothery—experts in human factors. Longer yellows causing more red-light running has never once been shown true. All research papers demonstrate the reverse.

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