In most problems in uniformly accelerated motion, two parameters are known and a third is sought. It is convenient, therefore, to obtain relations between any three of the four parameters. Equation 3-12 contains \( v_x, a_x, v_t, \) and \( t \), but not \( x \); Eq. 3-14 contains \( x, v_x, \) and \( t \) but not \( a_x \). To complete our system of equations we need two more relations, one containing \( x, a_x, \) and \( t \) but not \( v_x \) and another containing \( x, v_x, \) and \( a_x \) but not \( t \). These are easily obtained by combining Eqs. 3-12 and 3-14.

Thus, if we substitute into Eq. 3-14 the value of \( v_x \) from Eq. 3-12, we thereby eliminate \( v_x \) and obtain

\[
x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2.
\]

(3-15)

When Eq. 3-12 is solved for \( t \) and this value for \( t \) is substituted into Eq. 3-14, we obtain

\[
v_{x}^2 = v_{x0}^2 + 2a_{x}(x - x_0).
\]

(3-16)

Equations 3-12, 3-14, 3-15, and 3-16 (see Table 3-1) are the complete set of equations for motion along a straight line with constant acceleration.

### Table 3-1

**Kinematic equations for straight line motion with constant acceleration**

<table>
<thead>
<tr>
<th>Equation Number</th>
<th>Equation</th>
<th>Contains</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-12</td>
<td>( v_x = v_{x0} + a_xt )</td>
<td>( x, v_x, a_x, t )</td>
</tr>
<tr>
<td>3-14</td>
<td>( x = x_0 + \frac{1}{2}(v_{x0} + v_x)t )</td>
<td>( x, \checkmark, \times, \checkmark )</td>
</tr>
<tr>
<td>3-15</td>
<td>( x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2 )</td>
<td>( \checkmark, \times, \checkmark, \checkmark )</td>
</tr>
<tr>
<td>3-16</td>
<td>( v_x^2 = v_{x0}^2 + 2a_{x}(x - x_0) )</td>
<td>( \checkmark, \checkmark, \checkmark, \times )</td>
</tr>
</tbody>
</table>

A special case of motion with constant acceleration is one in which the acceleration is zero, that is, \( a_x = 0 \). In this case the four equations in Table 3-1 reduce to the expected results \( v_x = v_{x0} \) (the velocity does not change) and \( x = x_0 + v_{x0}t \) (the displacement changes linearly with time).

The curve of Fig. 3-7b is a displacement-time graph for motion with constant acceleration; that is, it is a graph of \( x = x_0 \) in which \( x_0 = 0 \). The slope of the tangent to the curve at time \( t \) equals the velocity \( v_x \) at that time. Notice that the slope increases continuously with time from \( v_{x0} \) at \( t = 0 \). The rate of increase of this slope should give the acceleration \( a_x \), which is constant in this case. The curve of Fig. 3-7b is a parabola since Eq. 3-15 is the equation for a parabola having slope \( v_x \) at \( t = 0 \). We obtain, on successive differentiation of Eq. 3-15,

\[
x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2
\]

or

\[
v_x = v_{x0} + a_xt,
\]

which gives the velocity \( v_x \) at time \( t \) (compare Eq. 3-12), and

\[
dv_x/dt = a_x
\]

the constant acceleration. The displacement-time graph for uniformly accelerated rectilinear motion will therefore always be parabolic.

**EXAMPLE 4**

You should not feel compelled to memorize relations such as those of Table 3-1. The important thing is to be able to follow the line of reasoning used to obtain the results. These relations will be recalled automatically after you have used them repeatedly to solve problems, partly as

3-9

**CONSISTENCY OF UNITS AND DIMENSIONS**