Uncertainty in the Yellow Change Interval

Abstract

The difference between legal and illegal when it comes to running a red light is not black and white. Engineering is never exact. Tolerance is routine for engineering. Yellow change intervals are no exception.

Up until the red-light camera, it was the responsibility of a policeman to personally enforce the local ordinances prohibiting red light running. Rarely is there a problem when a policeman enforces the law. The good policeman understands that the driver has a certain amount of guess work when deciding how to react to a yellow light. So the policeman allows a driver to run a red light up to some small amount of time, somewhere around one second.

Then things changed. Cities introduced red-light cameras. The red-light camera systems began enforcing the red light running laws, but these systems enforce the laws with zero tolerance. Camera systems expect drivers at all times and circumstances to make stop and go decisions with infinite accuracy.

Though the policeman comes closer to justice than the camera, neither policeman nor camera do the right thing. The right thing is for the engineer to calculate the tolerance, then tell law enforcement the required tolerance to give drivers. The value for the yellow change interval is the interval with its tolerance. Specifying a yellow change interval without its tolerance is a mistake according to the physical and mathematical sciences.

The method of computing the tolerance is called error propagation. Scientists, engineers and mathematicians have been using error propagation for centuries. It is the common and only method for calculating tolerances. Based in calculus, the technique is elegant. It readily shows how the uncertainties in the inputs to the equation affect the uncertainty in the overall value calculated from the equation. Note that tolerance, uncertainty and “error in the calculation” are synonyms.

A policeman usually gives about 1 second of tolerance. Many red-light cameras give drivers about 0.2 seconds. Some cameras give drivers 0.5 seconds. Some cameras give no tolerance.

The correct tolerance is around 2.5 seconds. This paper shows how to calculate the tolerance.

Engineering Requires Law Enforcement to Show Tolerance

Many engineers use the ITE yellow change interval formula. Because the inputs to the ITE formula are not precise, the yellow change interval is not precise. Approach speed, perception-reaction time, deceleration, grade of road are imprecise measurements. The engineer inputs these measurements into the ITE formula. By deriving the error propagation formula from the ITE formula then plugging in the uncertainties of the inputs, we then compute the uncertainty of the yellow change interval.

The engineer must share this uncertainty with law enforcement so that law enforcement grants drivers the proper tolerance according to the limits of engineering.

Unimpeded Straight-Through Movement Traffic – the ITE Equation

The ITE equation describes the kinematics of only straight-through unimpeded vehicles. Unimpeded through-movement vehicles approach at v_0. Once too close to the intersection to stop comfortably, the vehicles proceed toward the intersection at a speed greater than or equal to v_0. This case is the only case when the ITE equation gives sufficient distance and time for the driver to legally navigate the intersection. We will first derive the error propagation formula for the ITE equation.

(In the next section, we consider the error propagation formula for turning vehicles. The yellow change interval equation for turning vehicles is different.)

To make the math easier, we consider v_0 as a true constant and ignore the grade of the road. We consider only the uncertainties in perception-reaction time t_p and deceleration a.

1	Yellow Change Interval Function for Straight-Through Movement[1] $\boldsymbol{Y \geq t_p + \frac{v_0}{2a}}$
	Uncertainty in the Yellow Change Interval for Straight-Through Movement

2	$$\Delta \boldsymbol{Y} = \left\|\frac{\partial Y}{\partial t_p}\Delta \boldsymbol{t_p}\right\| + \left\|\frac{\partial Y}{\partial a}\Delta \boldsymbol{a}\right\|$$ Where: Δt_p is the uncertainty in the perception-reaction time. Δa is the uncertainty in the deceleration of a vehicle.
3	$$\Delta \boldsymbol{Y} = \left\|\Delta \boldsymbol{t_p}\right\| + \left\|-\frac{\boldsymbol{v_0}}{\boldsymbol{2a^2}}\Delta \boldsymbol{a}\right\|$$
4	$$\Delta \boldsymbol{Y} = \Delta \boldsymbol{t_p} + \frac{\boldsymbol{v_0}}{\boldsymbol{2a^2}}\Delta \boldsymbol{a}$$

The yellow change interval formula is not your usual scientific function. Normally a scientist or engineer applies error propagation to some formula whose measured input variables are *exact values*. An exact value has one and only one value in the universe. A true constant. The measurements for the constant yield a Gaussian distribution of values. The average of those measurements is the constant and the standard deviation is the constant's uncertainty. The yellow change interval formula is different. What appears to be constants, t_p and deceleration a, are not constants. They are not exact values. t_p and a are ranges of equally-valid values.

Because the values are ranges, the uncertainty is the sum of the absolute values of the partial derivatives of the function with respect to each measured input variable multiplied by the uncertainty of each input variable. Had the values represented constants and the measurements yielded Gaussian distributions, we would have used quadrature to propagate errors. As it is, we propagate the errors linearly.

Because the inputs are ranges, instead of using the standard deviation for Δt_p and Δa, we use 1/2 of the range of t_p and a. For example, we know that Δt_p has a range from about 0.5 seconds to about 3.5 seconds (AASHTO's range)[2, 3]. That range is 3 seconds long which makes the uncertainty Δt_p to be approximately $\pm$1.5 seconds.

The following are more facts to consider:

Perception/Reaction Time

1. Engineers treat perception-reaction time as a constant but it is not a constant.

2. Perception-reaction time is often expressed in two components: 1) perception time and 2) reaction time.

3. Perception time is the time a driver takes to perceive that the light turned yellow and to think about what action to take next.

4. Reaction time is the time it takes for a driver to move his foot to the brake pedal.

5. Perception time is a complex formula. Perception time is a function of the complexity of the intersection (AASHTO gauges the level of complexity in terms of "information bits"[2]), the distance the driver is from the intersection when the light turns yellow, the age of the driver, the alertness of the driver, the time of day, the weather, the vehicle's characteristics and many more variables[3].

6. Traffic engineers take measurements of the combined perception-reaction times. Engineers do not take measurements of perception time and reaction time separately.

7. Engineers restrict measurements of perception-reaction time to a subset of drivers. The subset is the set of drivers who stop. Engineers ignore the perception time of the set of drivers who go. Engineers measure a perception-reaction time by measuring the time the light turned yellow to the time the car's brake lights come on. Perception-reaction time is more accurately described as brake-response time.

8. The quickest value of perception-reaction time is around 0.5 seconds. The slowest value of perception-reaction is about 3.5 seconds.

9. Traffic engineers lowball the perception-reaction time by systematically assuming a simple low-information intersection. ITE uses 1.0 second. Most engineers adopt this value without studying the specifics of the intersection. Regardless of what engineers adopt, the fully-qualified value for t_p is 2.0 ± 1.5 seconds.

Deceleration

1. The average comfortable deceleration of passenger vehicles on dry pavement is about 10 ft/s^2.

2. The deceleration of the best performance commercial drivers with an empty tractor-trailer[3,4] is about 8.0 ft/s^2.

3. The Transportation Research Board measured the maximum safe and comfortable deceleration of passenger vehicles as 12.0 ft/s^2.

4. Deceleration changes in the rain or snow. Rain reduces the coefficient of friction on the pavement by 20 to 30 percent[5]. When a driver brakes on dry pavement at 10.0 ft/s^2, then pushes down his brake pedal to the same extent on wet pavement, he decelerates at 8.0 ft/s^2. Compensating for the rain requires an increase in the driver's perception-reaction time and requires that the vehicle's brakes-tires can exert a sufficient opposing force.

5. We use $\pm$ 2.0 ft/s^2 for the uncertainty in the deceleration in order to retain ITE's median deceleration of 10 ft/s^2. We are underestimating the uncertainty here but it is not that important to be precise about the imprecision. It is important only to get a ballpark figure.

Plug in Numbers

5	➢ The uncertainty in perception-reaction is $\pm\mathbf{1.5}$ seconds. ➢ The speed limit is 45 mph (66.2 ft/s). ➢ The deceleration is 10 ft/s^2. ➢ The uncertainty of the deceleration is about $\pm$2.0 ft/s^2. ➢ From equation 4 the uncertainty in the yellow change interval is: $$\Delta Y = \mathbf{1.5} + \frac{\mathbf{66.2}}{\mathbf{2(100)}}(\mathbf{2.0})$$

6	For a 45 mph speed limit, the uncertainty in the yellow light change interval for a driver who traverses unimpededly at the speed limit through the critical distance and into the intersection is: $$\Delta \boldsymbol{Y} = \pm \mathbf{2.2} \textbf{ seconds}$$
7	Using the middle of the range for perception/reaction time and deceleration, the proper way to express the yellow change interval for a 45 mph level road is: $$\boldsymbol{Y} = \mathbf{5.3} \pm \mathbf{2.2} \textbf{ seconds}$$ Engineers use ITE values for t_p and a thus lowballing Y to 4.3 seconds.

Mandatory Directive from the Traffic Engineer to the Police Regarding Unimpeded Straight-Through Traffic Movement

"Because of the imprecision in the computation of the yellow change interval, you cannot legally ticket a driver unless the driver runs the red light by at least 2.3 seconds into the red. This tolerance is the engineering requirement for straight-through movement drivers who do not decelerate en route into the intersection. If you ticket these drivers, you are violating engineering practices and subsequently MUTCD standard 4D.26 (03).[6]

A 5.3 second yellow interval seems long for a 45-mph level road. Yet commercial truck drivers, passenger vehicles travelling in bad weather and/or older drivers approaching a simple intersection may need more time than that. On the other end of the spectrum is the 20-year-old driving a Maserati in good weather. He may need only 3.0 seconds. All these various types of vehicles and drivers are *allowed* on the road. Each requires his own yellow change interval set by the laws of physics. The engineer must accommodate them all in order to safeguard the public from unjust financial injury.

Turning Traffic

To make the math easier, we consider only the uncertainties in t_p, a and v_i.

1	Yellow Change Interval Function for Turning Movement[1,7] $$\boldsymbol{Y \geq \frac{v_0\left(t_p + \frac{v_0}{2a}\right)}{\left(\frac{v_0 + v_i}{2}\right)}}$$ $\boldsymbol{v_i}$ is the velocity of the vehicle at the intersection stop bar.
2	$$\boldsymbol{Y \geq \frac{2v_0\left(t_p + \frac{v_0}{2a}\right)}{v_0 + v_i}}$$
3	Uncertainty in the Yellow Change Interval for Turning Movement $$\boldsymbol{\Delta Y = \left\lvert\frac{\partial Y}{\partial t_p}\Delta t_p\right\rvert + \left\lvert\frac{\partial Y}{\partial a}\Delta a\right\rvert + \left\lvert\frac{\partial Y}{\partial v_i}\Delta v_i\right\rvert}$$ Where: Δt_p is the uncertainty in the perception-reaction time. Δa is the uncertainty in the deceleration of a vehicle. Δv_i is the uncertainty in the intersection entry speed of a vehicle.
4	$$\boldsymbol{\Delta Y = \left\lvert\frac{2v_0}{v_0 + v_i}\Delta t_p\right\rvert + \left\lvert\frac{{v_0}^2}{a^2(v_0 + v_i)}\Delta a\right\rvert + \left\lvert\frac{\partial Y}{\partial v_i}\Delta v_i\right\rvert}$$

5	Apply the derivative of the reciprocal law to the last term. $$\frac{\partial Y}{\partial v_i} = \frac{\partial(1/f)}{\partial v_i} = -\frac{1}{f^2}\left(\frac{\partial(f)}{\partial v_i}\right)$$
6	$$f = \frac{v_0 + v_i}{2v_0\left(t_p + \frac{v_0}{2a}\right)} = \frac{v_0}{2v_0\left(t_p + \frac{v_0}{2a}\right)} + \frac{v_i}{2v_0\left(t_p + \frac{v_0}{2a}\right)}$$
7	$$\frac{\partial(f)}{\partial v_i} = \frac{1}{2v_0\left(t_p + \frac{v_0}{2a}\right)}$$
8	$$\Delta Y = \left\lvert\frac{2v_0}{v_0 + v_i}\Delta t_p\right\rvert + \left\lvert\frac{{v_0}^2}{a^2(v_0 + v_i)}\Delta a\right\rvert + \left\lvert\left(\frac{2v_0\left(t_p + \frac{v_0}{2a}\right)}{(v_0 + v_i)^2}\right)\Delta v_i\right\rvert$$

9	➢ **The uncertainty in perception-reaction is ± 1.5 seconds.** ➢ **The speed limit is 45 mph (66.2 ft/s).** ➢ **The intersection entry speed is 20 (29.4 ft/s).** ➢ **The range in the intersection entry speed is 10 mph to 35 mph.** ➢ **The uncertainty in the intersection entry speed is $\pm$12.5 mph (18.4 ft/s).** ➢ **The deceleration is 10 ft/s^2.** ➢ **The uncertainty of the deceleration is about $\pm$2.0 ft/s^2.** ➢ **Therefore from equation 8, the uncertainty in the yellow change interval is:** $$\Delta Y = \left\|\frac{2(66.2)}{66.2 + 29.4}(1.5)\right\| + \left\|\frac{66.2^2}{(10)^2(66.2 + 29.4)}(2.0)\right\| + \left\|\left(\frac{2(66.2)\left(2 + \frac{66.2}{2(10)}\right)}{(66.2 + 29.4)^2}\right)(18.4)\right\|$$ $$\Delta Y = \|2.1\| + \|0.9\| + \|0.7\|$$
10	$$\Delta Y = \pm 3.7 \text{ seconds}$$
11	Using the middle of the range for perception/reaction time and deceleration, the proper way to express the yellow change interval for a 45 mph level road for turning movement is: $$Y = 7.4 \pm 3.7 \text{ seconds}$$ Because engineers' current practice is to misapply the straight-through movement formula to turning movements, yellow change interval are extremely short. States like Florida will set Y to 4.3 seconds. At its red light cameras intersections, Florida is currently increasing Y to 4.7 seconds. In Arizona, California, North Carolina and Virginia, engineers also plug in the wrong numbers into the wrong formula and set Y to 3.0 seconds.

Mandatory Directive from the Traffic Engineer to the Police Regarding Turning Traffic Movement

"Because of the imprecision in the computation of the yellow change interval, you cannot ticket a driver unless the driver runs the red light by at least 3.4 seconds into the red. This tolerance is the engineering requirement for left and right turning drivers. U-turns require more tolerance. If you ticket these drivers, you are violating engineering practices and subsequently MUTCD standard 4D.26 (03).[6]

A 7.4 second yellow interval seems long for a 45-mph level road. Yet commercial truck drivers, passenger vehicles travelling in bad weather and/or older drivers approaching a simple intersection may need more time than that. On the other end of the spectrum, the 20-year-old driving a Maserati in good weather may need only 4.0 seconds. All these various types of vehicles and drivers are *allowed* on the road. Each requires his own yellow change interval set by the laws of physics. The engineer must accommodate them all in order to safeguard the public from unjust financial injury.

References

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